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On the reversal of the rotational momentum of Earth: a derivation and analysis of the Herodotus equation

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1 Introduction

Ancient texts contain statements about the change of the year length in days, requiring adjustments of the calendars, and the change of the location where the Sun rises and sets, implying a change of the direction of the rotational momentum of the Earth, more precisely a reversal if the positions of sunrise and sunset would exactly interchange. Mechanisms that may change the axis direction have been recently considered. Changes due to impact with a sufficiently large asteroid or comet have been studied by Barbiero [1]; changes due to the torque by a sufficiently large body passing near the Earth have been considered by Woelfli and Baltensperger [2]. Changes of the orbital parameters, hence of the length of the year, have been considered by Brunini [3] for the case of an impact. To our knowledge changes in both the rotation and revolution parameters, that are expected after this type of interaction, have never been modelled mathematically. Here we provide under simplifying assumptions such an analysis.

2 The Herodotus equation

Our analysis of the problem is based upon the following simplification:

1. we disregard the effects of the interaction over the body that is responsible for the event; this neglect may be acceptable in the case the change is triggered by a cometary/asteroid type impact of a kilometer size, whose momentum is order 10^{-12} of that of Earth, but is probably leading to large errors in the case the event is due to the close passage of a large body

2. we disregard the effects on the other bodies of the solar system

3. we consider the Earth as a rigid spherical neutral body, neglecting for instance thermal effects, differential dynamical effects on the different layers of Earth and so on

4. we assume that the Earth revolves, before and after the event, on the ecliptic plane with a rotation axis perpendicular to such a plane and on a circular orbit

5. we consider the case when the direction of the Earth rotational momentum is not changed. Thus sunset and sunrise points may interchange as well as the austral and boreal sky. Autumn and spring, summer and winter, may also interchange, a fact important at the medium-high latitudes, but possibly not important for the low latitudes.

Let m and M be respectively the mass of Earth and Sun, let r and R be respectively the radius of Earth and of its orbit before the reversal. Let ω and η be respectively the scalar values representing the rotational velocity and revolutionary velocity of Earth before the reversal (i.e. the components of the velocity vactors orthogonal to the ecliptic plane); after reversal such values will be $-\omega+\Delta\omega$ and $\eta+\Delta\eta$. We write two equations, based on the conservation of momentum and energy, that will allow the determination of $\Delta\omega$ and $\Delta\eta$. One should note that if the agent whose action leads to reversal should be considered, the obtained equations would not be sufficient to provide well defined solutions (in other terms: one should consider the interaction in detail via suitable dynamical equations).

The momentum due to the orbital revolution is

$$M_1 = mR^2\eta$$

The momentum due to the rotation is

$$M_2 = (2/5)mr^2\omega$$

By the third law of Kepler we have, with $T = 2\pi/\eta$ the period of revolution of Earth, G the gravitational constant and as usual disregarding the addition of m to M

$$T^2/R^3 = const = (4\pi^2)/(GM)$$

implying

$$R = (GM/\eta^2)^{1/3} \tag{1}$$

We need the above expression for R in the equations relating to the state after the reversal. The relation must also be satisfied by the initial values of η and R.

The equation for conservation of momentum is obtained simply by summing up the two momenta, since in our approximation they are parallel vectors

$$R^2\eta + (2/5)r^2\omega = (GM)^{2/3}(\eta+\Delta\eta)^{-1/3} + (2/5)r^2(-\omega+\Delta\omega) \tag{2}$$

From the above we obtain, by defining

$$z = \eta + \Delta\eta$$

and taking (1) into account, we get

$$\Delta\omega = 2\omega + 2.5(GM)^{2/3}(\eta^{-1/3} - z^{-1/3})/r^2 \tag{3}$$

The initial kinetic energy due to rotation is given by

$$E_1 = mr^2\omega^2/5$$

while the kinetic energy due to orbital revolution is given by

$$E_2 = mR^2\eta^2/2$$

The potential energy associated to the revolution is given by

$$V = -GmM/R$$

The energy conservation equation is therefore, omitting the common term m and letting $H = GM$

$$r^2\omega^2/5 + R^2\eta^2/2 - H/R = r^2(-\omega + \Delta\omega)^2/5 + -H^{2/3}z^{2/3}/2 \tag{4}$$

or, observing that the kinetic energy is exactly half the potential energy in absolute value

$$r^2\omega^2/5 - R^2\eta^2/2 = r^2(-\omega + \Delta\omega)^2/5 - H^{2/3}z^{2/3}/2 \tag{5}$$

Substituting now in Equation (5) the value of $\Delta\omega$ given by (3) we get a nonlinear algebraic equation for $\Delta\eta$ that may be solved numerically. By letting $y = z^{-1/3}$ the energy equation can be written as follows

$$\alpha_0 + \alpha_1 y + \alpha_2 y^2 - \alpha^*/y^2 = 0 \tag{6}$$

where $\alpha_2 > 0$, $\alpha^* > 0$, implying that the right hand side is positive for large y, negative for sufficiently small positive y, so that there is at least one real solution (actually, at least two, the equation being reducible to a quartic polynomial without the linear term).

By letting $y = z^{1/3}$ we can also write the energy equation as a quartic polynomial without the cubic term, of the following form

$$\beta_0 + \beta_1 y + \beta_2 y^2 + \beta_4 y^4 = 0 \tag{7}$$

where, having used (1) to eliminate H

$$\beta_0 = -5R^2\eta^{5/3}/4$$

$$\beta_1 = 5R^2\eta^{4/3}/2 + r^2\eta^{1/3}\omega$$
$$\beta_2 = -5R^2\eta/4 - r^2\eta/2 - r^2\omega$$
$$\beta_4 = r^2\eta^{1/3}/2$$

We call equation (7) the *Herodotus equation*. Our numerical solution has used such equation. Notice that for r going to zero eq. (7) approximates in the limit the quadratic equation $y^2 - 2\beta y + \beta^2 = 0$, where $\beta = \eta^{1/3}$, with solutions $y = \beta$. It is easy to verify that $y = \beta = \eta^{1/3}$ is a solution also of (7), corresponding to $\Delta\eta = 0$ and $\Delta\omega = 2\omega$, hence this is the solution corresponding to the initial unchanged state, or to an even number of reversals. From this observation is is also easy to factorize explicitly the above polynomial into the product of $y - \eta^{1/3}$ by a cubic polynomial.

The numerical results are presented and discussed in the next sections. The following quantities are of interest:

1. the length of the day, given by

$$t_1 = 2\pi/(-\omega + \Delta\omega)$$

2. the length of the year, given by

$$t_2 = 2\pi/(\eta + \Delta\eta)$$

3. the number D of days in the new year, given by

$$D = t_2/t_1$$

4. the number h of hours in the new day, given by the following formula, assuming that time is measured in seconds

$$h = t_1/(3600)$$

Previous to the analysis of the numerical solution, we should make the following observations:

1. the solution does not depend on m, which certainly is an effect of having disregarded the change of momentum and energy on the external body that triggered the transition, and partially of having neglected m in Kepler's formula

2. it does not depend explicitly on G; recall the intriguing fact that the value of G is still very poorly known, only to three digits

3. the solution depends, by inspection of the coefficients of the quartic, on the initial values of ω and η, on the radius r of the body and on the radius R of its orbit (notice that R and η are related by the Kepler's equation (1)).

4. by the symmetry properties of the equation it is clear that $\Delta\eta = 0$ and $\Delta\omega = 2\omega$ are solutions, corresponding to the trivial case of no change in the initial state (e.g. by an even number of reversals)

5. again by the symmetry it is clear that if $\eta + \Delta\eta$ and $-\omega + \Delta\omega$ are solutions corresponding to given ω and η, so are ω and η solutions corresponding to given $\eta + \Delta\eta$ and $-\omega + \Delta\omega$.

3 Numerical solution of the equations

We have solved equation (7) for two cases of interest. The first is the case where Earth, in its present orbit, would be subject to a reversal. The second is the case of another reversal following the first one.

The numerical values for the parameters in the equation are the following, in CGS units

$$M = 2 * 10^{33}$$
$$R = 1.497766 * 10^{13}$$
$$r = 6.3 * 10^{8}$$
$$\omega = 2\pi/(24 * 60 * 60)$$
$$\eta = 2\pi/(365.24 * 24 * 60 * 60)$$
$$G = 6.66 * 10^{-8}$$

Our numerical results have been obtained using DERIVE 5 with 50 digits of precision. Notice that with 10 digits two complex solutions are obtained, albeit with a very small imaginary term. We also used MAPLE 8, noticing that it requires higher precision than DERIVE 5 to obtain the same solutions.The same solutions were also obtained by prof. Waldvoegel, of ETH, Zurich, who used MATLAB, and prof. Brugnano, of the university of Florence, who used the Trigiante-Brugnano algorithm for determining roots of polynomials.

The four solutions have the following features:

1. two solutions, one positive and one negative, imply extremely high value for η, e.g. of the order of hundred of revolutions per second, i.e. an increase of eight orders of the revolution speed; they are unacceptable within the planetary context. It remains an interesting open questions whether they might have a meaning within some possibly "exotic" physical context

2. the theoretically known solution $y = \eta^{1/3} = 5.839324324410^{-3}$, perfectly satisfies the equation, giving exact zero residual or extremely small residual

3. the fourth solution implies a small change in η and ω, leading to interesting changes in the duration of the day and of the year. The computed numerical value for y is $y = 5.83932131310^{-3}$. The change in η leads to a change in R of about 150.000 km, one thousandth of the distance Earth to Sun.

Using the fourth solution in the computation of the number of days in the year and hours in the day, gives 363.236 days in the "new" year, with a day of 24.13 hours, hence a change of about two days in the year and of about 8 minutes in the day; thus the relative variations in both rotation and revolution angular speeds are similar.

Solving the equation from the new state (i.d. changing the parameters ω, η, R, with the Kepler equation being satisfied) provides, using of the four solutions the one of physical interest, a year with 365.236 days and a day with 24.002 hours, i.e. virtually the state before the reversal, as theoretically expected (the small difference is due to the fact that the numerical va;lues of the used parameters do not have enough digits to exactly satisfy the Kepler's equation for the initial state).

The above numerical results lead to the following conclusions:

1. the change in the duration of the year is quite important. It would lead within few years to the need of changing the calendar, in particular the need to determine how many days would be in the new year, and in which day the new year would start. This fact may therefore be the explanation of the obsession with the calendar of many ancient people, especially in the second and third millennium BC, and of the need of constructing special structures to determine the solstice or equinox days

2. the second reversal returns the system to the state before the first reversal, apart from changes that may be an effect of numerical error. It thus appears, within the limits of the present model, that planet Earth may have oscillated between two close orbital/rotational states. The average value of the length of the year in the two states is about 364.1. It is interesting to note that 364 = 7 * 52, numbers associated with the days in the week and the weeks in the year, of sacral relevance for ancient peoples, 52 especially playing an important role in the Mayan calendar. Also, the calendar of the Essenes had 364 days, a fact quite puzzlying for interpreters

3. the change in the distance Earth to Sun implies a negligible reduction in the amount of radiation from the Sun , about one millionth, in agreement with the statement in the ancient documents that no significant climatic variation was observed after the event.

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2 - W. Woelfli, W. Baltensperger, A possible explanation for Earth's climatic changes in the past few million years, http://xxx.lanl.gov//abs/physics/9907033

3 - A. Brunini, Orbital evolution of the terrestrial planets as a result of close encounters and collisions with planet-crossing asteroids, Planet. Space Sc. 41, 747-751, 1993

4 - Jibril Mandel Khan, private communication (prof. Mandel is a leading scholar of the Quran)

Appendix 1: some ancient references to changes in the sunrise-sunset points

Herodotus (Book II, 142) refers that priests told him that *since Egypt became a kingdom four times in this period the Sun rose contrary to his wont; twice he rose where he now sets, and twice he set where he now rises. Egypt was in no way affected by these changes; the production of the land and of the river remained the same; nor was there anything unusual either in the diseases or the deaths*. It should be noticed that this remarkable statement is not included in the analytic index of Herodotus edition by Everyman, 1992!

Pomponius Mela (De Situ Orbis, I, 59, ed. Soc. Edit. Belles Lettres, Paris, 1988) gives essentially the same information as Herodotus, but apparently in an independent way: *mandatumque litteris servant, dum Aegyptii sunt, quater cursus suos vertisse sidera ac solem bis iam occidisse unde nunc oritur* (it is written in their documents that since Egyptians exist, four times the stars changed their course and the Sun twice settled in the past where it now rises).

In Quran, Sura 70, verse 40, Allah is referred to as *Lord of Orients and Occidents*. This verse, see [4], has always been an enigma to Quran interpreters, being usually considered as a poetical utterance.

Appendix 2: a tentative dating of the reversals claimed by Herodotus

A reversal would be noted by unusual movements in the celestial bodies and would most probably be also associated with high tides, possibly of catastrophic extent.

One such event is certainly seen in the episode of the sun standing in the sky for a long time when Joshua had entered Canaan and was following some enemies. Since Canaan is a high land (according to the Lebanese historian Kamal Salibi it was not Palestine but the highland in SW Arabia now called Asir) no high tides from the sea were reported in the Bible. But the Joshua event took place 40 years after Exodus, the time possibly of the Ogyges Flood, that devasted the Mediterranean and probably killed the Egyptian Pharaoh Apoph, the Agog of Bible, the first Hyksos Pharaoh according to the chronology of several nonstandard Egyptologists. We know from Manetho that the Egyptian calendar was modified some time later by another Hyksos Pharaoh.

Another reversal is suggested by another statement in Manetho, claiming that at the time of a certain pharaoh of the Ancient Kingdom the moon had unusual movements. Study of widespread reconstruction of megaliths in northern Europe around 2300 BC, a date compatible with the time of the Manetho pharaoh, suggests that also this event was associated with calendrical changes.

Finally it is known that more than a dozen calendars were modified circa 700 BC, incuding the one in Rome by Numa Pompilius. We are not aware now of reports of unusual movements of celestial bodies that would suggest a reversal.

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