ABSTRACT - The joint behaviour of equity premiums and credit spreads on securities issued by the same company provides a direct statistical evidence of the degree of efficiency of equity and fixed income markets, whose participants are expected in the long term to provide a common risk assessment.

Increasing interest in the financial industry is attracted both for financial engineering and trading purposes, by the potentials offered by contracts with equity and fixed income components.

Increased liquidity in the credit default swaps (CDS) market, on the other hand, provides new grounds for fixed income analysis based on the statistical study of theoretical versus actual spread movements.

In the paper we analyse the statistical relationship between CDS spreads, stocks implied volatility and theoretical spreads generated by an application of Merton seminal structural default model. A measure of price discrepancies is also proposed, based on the difference between theoretical and actual spread behaviours, leading to an application of relative value analysis in the fixed income market.

We also test the dependence of credit spreads behaviour on the volatility of the associated equity returns and the relationship over time between theoretical and observed CDS spreads. The analysis is applied to six large corporations and back-tested over the 2002-2003 period. Ford and General Motors are considered for the sector of Automotives, Deutsche Telekom and France Telecom for Telecommunications, Endesa and RWE for the sector of Utilities.

We show that implied volatility movements drive significant spread movements: both theoretical and actual spreads follow volatility patterns. Furthermore, the impact on the theoretical spreads provides an accurate proxy of forthcoming movements in actual spreads. This general results applies in particular to companies with a sufficient degree of risk (i.e. leverage), while poor evidence can be collected for companies relatively free of default risk.

KEYWORDS – Credit risk, Structural default model, Relative value analysis, Credit default swaps, implied volatility.
1 INTRODUCTION

Increasing liquidity in the credit default swaps (CDS) market provides a much cleaner environment in which to study the statistical properties of credit spreads. In what follows we extend an approach originally conceived for credit risk modelling to assess company-specific mis-pricing indicators. The analysis is built on the comparison of data generated by an application of Merton’s seminal structural default [Merton, 1977] model against CDS spreads (on the pricing of CDS see [Duffie, 1999] and [Hull and White, 2000]). We test the dependence of credit spreads on the volatility of the associated equity returns and the relationship over time between theoretically generated spreads and actual CDS spreads. The general aim of the article is to test the potential of the structural approach to default risk estimation as reference model for relative valuation of fixed income securities. Two such applications are also briefly described.

We support our conclusions relying on the following statistical analysis:

- We clarify in a static framework, how theoretical spreads are implied out using the Merton approach, and the general structure of the approach;
- We introduce a measure of price discrepancy based on the relationship between theoretical and actual market spreads, described by the spreads quoted in the Credit Default Swap market, and
- Test the opportunities offered by this approach in the construction of strategies in the fixed income market. In terms of *cheap-dear* analysis and *convergence trades* effectiveness.

Observe that the developed theoretical model is not intended to provide accurate *fair* valuation of market spreads. There is already extensive evidence of a structural mis-valuation of market spreads by this approach [Campbell and Taksler, 2003], [Cooper and Davydenko, 2003]. Rather we test whether normalised spreads, i.e. spreads adjusted by their mean and volatility can be used for relative valuation purposes.

The analysis has been applied to six large corporations and back-tested over the 2002-2003 period. Ford and General Motors are considered for the Automotive sector, Deutsche Telekom and France Telecom for Telecommunications, Endesa and RWE for the sector of Utilities.

For each such company we apply Merton’s approach – see § 2 for details – to derive theoretical implied credit spreads, using as inputs, among others, the equity implied volatility, and compare the resulting values with observable CDS spreads.

Results are extremely interesting. In particular, according to our model, implied volatility movements drive significant spread movements and both theoretical and actual spreads follow these volatility patterns. Furthermore, the impact on the theoretical spreads provides an accurate proxy of forthcoming movements in actual spreads. These general results apply mostly to companies with a sufficiently high degree of leverage while the relationships are weaker for companies relatively free of default risk.

In the normalised set-up theoretical spreads show higher volatility than actual spreads and provide alternatively upper and lower bounds to actual spread movements: this evidence is used to introduce a measure of mis-pricing moving around 0 with actual spread movements following mis-pricing indications.

The article structure is as follows. In section 2 we analyse the assumptions and main features of Merton’s structural default model [Merton, 1974]. In section 3, we study the statistical interaction between theoretical and market spreads and analyse their relationship with equity implied volatility. Finally in section 4 the potentials offered by this framework in terms of relative value trading are tested.
2 THE MERTON STRUCTURAL DEFAULT MODEL

High and increasing leverage, measured by the debt-to-equity ratio implies increasing interest payments, potentially lowering earnings expectations and reducing expected returns on the equity investment. Typically if a company needs to increase financial resources to meet short-term obligations both the cost of debt and the cost of equity capital will increase penalising equity and bondholders alike.

The process, as is well known, does not need to be symmetric. Increasing earnings expectations are likely to induce an increase of the expected return on the equity investment and a decrease of the cost of equity capital, while the cost of debt remains the same.

Merton’s method is generally considered to provide a coherent way to extract information on theoretical credit spreads and default probabilities from information about the company’s leverage, the company’s value and the volatility of the company’s value. Stock price volatility can be regarded as the crucial variable of the approach.

The option-pricing corporate default model exploits the analogy between holding the equity of a company and going long a call option whose underlying is the company’s total asset value. The strike price is given by the nominal value of the outstanding debt.

Figure 2.1
Equity holding as a long Call option

In this framework the Black and Scholes option formula [Black and Scholes, 1973] can be used to derive a credit spread/default probability from the company’s equity value and volatility with a maturity given by the maturity of the outstanding debt.

Just to expand on this point a company’s assets can be funded through a combination of equity and debt. If the assets fall in value below the level of the debt, in principal, creditors have the right to claim the nominal value of their credit. In this situation, on the other hand, equity holders do not benefit before all debt-holders are paid off.

The asymmetry is reflected in the resulting payoff patterns: equal to a European short-put option for a debt-holder and a long-call option for an equity-holder. This is the evidence used by Merton to propose an option-valuation framework: from a given debt-equity structure and assets value volatility, the likelihood of a company’s default can be derived introducing
as strike price of the option (put or call), the outstanding nominal value of the debt (see figure 2.1).

2.1 The mathematical model

Suppose a very simple liability structure formed by equity and a zero-coupon bond issue maturing at time $T$. Let $V_t$ and $E_t$ denote the value of the company’s assets and, respectively, equity at time $t \in [0,T]$, $D$ the amount of debt to be repaid at time $T$ and $\sigma_V$ and $\sigma_E$ the volatility of the assets value and the equity value, respectively. To make clear the concept of default put forward by Merton, consider the following: If $V_T < D$, then is rational for the company to default on the debt at time $T$. In this case the value of the equity is 0. If instead $V_T > D$, the company should make the repayment at time $T$ and the value of the equity at this time is $V_T - D$. The resulting payoff for the firm’s equity at $T$ is thus $E_T = \max(V_T - D,0)$. This shows that the equity can be treated as a call option on the value of the assets with a strike price equal to the repayment required on the debt. We can thus apply the option pricing formula, from which the value of the equity today comes out the well-known Black and Scholes formula:

$$E_0 = V_0 N(d_1) - D e^{-rT} N(d_2)$$  \hspace{1cm} (2.1.1)

In (2.1.1): $N$ is the cumulative standard normal distribution and $r$ is a constant risk-free interest rate quoted at the current date for the debt maturity $T$. The value of the equity is decomposed in a weighted portfolio long the assets and short the discounted debt value. The weights are determined by the two percentiles $d_1 = 1/\sigma_V \sqrt{T} \left[ \ln(V_0 / D) + (r + \sigma_V^2 / 2)T \right]$ and $d_2 = d_1 - \sigma_V \sqrt{T}$.

The risk-neutral probability that the company will default on the debt at maturity is now $N(-d_2)$: this is the probability that at option expiry, the assets value will be below the debt value. To calculate this value we need $V_0$ and $\sigma_V$. The point is that neither of these is observable. However, we can observe $E_0$ and estimate $\sigma_E$. In our implementation we use the implied volatility estimated in the option market.

From Ito’s lemma we have

$$\sigma_E E_0 = N(d_1) \sigma_V V_0$$  \hspace{1cm} (2.1.2)

Equations (2.1.1) and (2.1.2) provide a pair of simultaneous equations that need to be solved for $V_0$ and $\sigma_V$. The resulting current value of the debt is $V_0 - E_0$.

From the debt value, the implied yield to maturity can be computed and, given the prevailing risk-free rate, the corresponding credit spread for that maturity calculated by subtraction, allowing comparison with the observable credit spread in the market.

The distance to default, given by $\frac{V_0 - D}{V_0 \sigma_V}$, can at this point be computed (see on the relevance of the $D2D$ the remarks in [Kealhofer, 2003a and 2003b]).
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The variable $V_0\sigma_Y$ is approximately equal to the size of one standard deviation move in the current assets value. The distance to default is thus an approximate measure of the number of standard deviations between the current value of the assets and the amount owed at maturity.

The system given by equations (2.1.1) and (2.1.2) is solved through the following fixed-point procedure.

We first rearrange the system to obtain more numerically stable functions (the function on the right of (2.1.1), being a difference between two quantities of same sign and similar size, may indeed lead to numerical errors):

$$V_0 = \frac{D e^{-rT} N(d_2) + E_0}{N(d_1)} \quad (2.1.3)$$

and

$$\sigma_Y = \frac{\sigma_Z E_0}{D e^{-rT} N(d_2) + E_0} \quad (2.1.4).$$

Which is a fixed-point equation of type $(V_0, \sigma_Y) = G(V_0, \sigma_Y)$.

To solve numerically this system we simply compute the iterations $G^n$ of function $G$ from an arbitrary (but sensible) starting point, until our error function, given by $\left|V_0^n - G_1(V_0^n, \sigma_Y^n)\right| + \left|\sigma_Y^n - G_2(V_0^n, \sigma_Y^n)\right|$ became less than $10^{-6}$, or until we reach iteration number 1000. In the latter case the function returned an error.

The solution of the system allows the definition of the current debt value as:

$$D_0 = V_0 - E_0 \quad (2.1.5)$$

from which the implied credit spread for the given maturity and the recovery rate associated with the given default probability, can be computed:

$$\pi_{0,T} = 10000\left(\frac{D(T)}{D(0)}\right)/T - r^f_{0,T} \quad (2.1.6)$$

$$\text{RecRate} = 1 - \left[1 - \frac{D(0)}{D(T)e^{-r_{0,T}}}\right]/\left(N(-d_2)\right) \quad (2.1.7)$$

Equation (2.1.6) defines the credit spread in basis points.

In § 3 we analyse the relationship between the spread computed in (2.1.6) – model generated, theoretical – with the 5Y CDS spreads observed in the market over a two-year period.

2.2 Model summary

Here below the information flows underlying the model implementation are displayed in order to summarise the model dependencies.
Several key financial variables are combined to derive an estimate of the current firm value and thus, by difference with the company market cap, the implied debt value needed to estimate the default probability and the recovery rate.

Several simplifications are needed in order to apply in practice this framework (see [Black and Scholes, 1973], [Geske, 1977], [Merton, 1974]):

- The company’s liability structure needs to be particularly simple. Only stocks and bonds are present. Changes in assets value need to be fully reflected in corresponding changes of equity and bond prices.
- All the debt matures at one point in time: this is the exercise time of the embedded option.
- The value of the equities and the assets follows a stochastic evolution described by a geometric Brownian motion.
- The market of corporate securities is free of arbitrage.

Input variables of Merton’s pricing model are: the stock price volatility and market value, the current nominal debt, the risk-free return for the average maturity of outstanding debt.

The outputs are the company market value and its volatility, the debt market value and the associated default information. Namely the default probability and the recovery value – the value of the debt recovered upon default – the implied credit spread for the assumed (average) maturity and the so-called distance-to-default (D2D) a measure of the number of standard deviations of the asset value away from default.
Theoretical spreads are derived within Merton’s approach by first computing the current debt value as a difference between the company’s total value and the market value of the equity capital and then by subtracting from the resulting implied yield the risk free rate.

2.3 Important theoretical relationships

The model allows a direct comparison between several key corporate variables. As clearly described in figure 2.2.1 and the model mathematical structure, Merton’s approach can be regarded as a mapping between five input variables, known at the current date and the company value and default information. The only input variable that typically requires statistical estimation is the stock return volatility: this not the case if the implied volatility is used instead. This is indeed the volatility measure considered here below.

We complete the description of the model by focusing on a set of relationships that clarify the assumptions underlying the proposed approach. Namely, between:

- Stock price volatility and company value volatility
- Credit spread and stock price and volatility
- Implied bond price and stock price and volatility
- Implied stock price and credit spread and default probability
- Stock price, credit spread and leverage.

The first relationship is behind the structural concept of default: this is assumed to occur when the asset value falls below the outstanding debt; the higher the firm volatility the higher the probability of default and the greater the option premium to gain protection. Observe that the very same reasoning is behind an increase of the spread to be paid to buy default protection via a CDS. Given the company value and the equity value we can by difference estimate the debt current implied value and thus the credit spread and implied bond price. Finally we are interested to understand the critical relationship between the leverage and the stock price and credit spread.

2.3.1 Equity and firm volatility

Equation (2.1.4) clarifies the relationship between equity and company volatility: we present below for a range of stock volatility values the estimated company volatility for a representative highly levered company.

Figure 2.3.1.1
Stock and Company volatility
As the equity volatility increases so does the firm value volatility: the rate of increase is exponential and for very high implied volatility inputs the company volatility increases more than proportionally, leading to a sudden reduction of the distance to default and respectively an increase of default probability.

2.3.2 Implied credit spread and bond price versus stock price and volatility

The spread defines the excess return – premium -- required for an investment in a risky debt instrument. The described model does provide a unique insight into the relationship between cost of debt and cost of equity capital. Consider the sensitivity of the implied spread to the equity volatility: the risk associated with an equity investment in the given company.

Figure 2.3.2.1 shows a 3D plot whose entries are the implied spread, the stock price and its volatility.

The model assumes an exponential relationship between credit spreads and equity volatility: the lower the price the stronger the volatility effect. Conversely the spread decreases for higher stock prices as expected.

A similar impact is found on the bond price implicit to such theoretical spread: the relationship can again be implemented for given stock price and volatility by implying out the price of one bond maturing at T from the estimated debt theoretical value. The bond price will decrease as the stock volatility increases and for increasing stock price a smaller reduction is recorded.

The option formula can be inverted in order to recover for given implied spread and default probability what the (implied) stock price should be. In figure 2.3.2.2 we report on the same
plot this interesting relationship: for increasing implied spread, thus increasing default probability and decreasing debt value, the stock price will tend to decrease. The sensitivity to the implied spread, maintaining the default probability fixed, is almost zero for low default values and becomes slightly negative for high default values.

**Figure 2.3.2.2**
Implied stock price and credit spread and default probability

In what follows we are interested in the first set of relationships from Figure 2.3.2.1, analysing the behaviour of the theoretical spread as a function of the stock price volatility. The second set of relationships can support the analysis of theoretical versus actual stock price movements in periods of stable volatility and is typical of equity research.

### 2.3.3 Stock price, credit spread and leverage

A relevant relationship, worth further analysis, is the one, for fixed debt value and equity volatility, between stock price, leverage and implied spreads. Stock price movements induce on one hand changes in market capitalization and for given debt value, corresponding changes in the leverage measured by the ratio between $D(t)$ and $E(t)$, and on the other hand changes in the credit spreads as explained above. At any point in time, the stock price reflects a given price yield: the dynamics of this yield relative to the credit spread is the fundamental driver of the leverage behavior.

For fixed risk free rate and time to maturity we can map the implied spread on the leverage for the current stock price and construct the corresponding theoretical leverage for the given credit spread and stock price pair. This is reported in figure 2.3.3.1.
The inputs are: the credit spread and the stock price for the X and Y axes, the risk free rate and the outstanding debt which, as the number of issued equities, are kept fixed. We can in this way recover the leverage as a ratio between the debt value associated with that risk free rate and the corresponding spread and the equity value induced by the corresponding stock price, given the number of equities.

The figure shows that as the stock price increases for constant credit spread the leverage tends accordingly to decrease at an exponential rate. Similarly for fixed stock price the leverage tends to decrease for increasing credit spread, but at a low linear rate. This behavior is consistent with a generally recognized evidence in market practice: the strong correlation between credit spreads, equity volatility and yields during periods of increasing market uncertainty and spread tightening, and the tendency of equity yields to react slower to turns during periods of spread easing.

Questions such as the impact on the cost of equity and debt capital associated with a leverage increase can in this framework receive an answer.

3 STOCK PRICE VOLATILITY, THEORETICAL AND CDS SPREADS

Merton’s approach provides a unique insight into the theoretical relationship between equity value and volatility and default probability and credit spreads. A recognised drawback of the approach, however, is that due to the necessary simplifying assumptions and in absence of specific additional corporate information, the approach leads to a severe underestimation of observable credit spreads and inaccurate assessment of the company default probability. Several approaches have been proposed to overcome this shortcoming (see [Campbell and Tak-
Taksler, 2003],[Geske, 1977],[Kealhofer, 2003a]) but no clear progress has yet been achieved.

The ability of the structural approach to track continuously changing credit exposures, however, has been widely assessed, leading to a widespread adoption of this modelling technique (see [Consigli, 2004],[Hull et al, 2004],[Kealhofer, 20003a and 2003b]). The method is robust with respect to credit risk ranking and as shown below can also be fruitfully applied to relative valuation purposes.

It is worth recalling that the model described in section 2 provides the canonical reference for the development of credit risk models based on this structural approach: the only required market variable is the company’s equity. KMV, the US company recently acquired by Moodys, is the most noticeable example of a successful story of application of this approach. Here the behaviour of the credit spread and the distance to default are turned into a default probability and rating group transition probabilities by introducing a set of class boundaries, whose crossing is associated with a rating transition. The following figure describes the result of an estimation procedure of this type on a representative highly levered company.

Figure 3.1
An application of the structural approach to credit risk modelling

The company value pdf at the 1Y horizon is generated by an application of the lognormal model. The boundaries for rating transitions are externally imposed to fit exogenous transition probabilities. Each sample path of the company value has an associated current debt value and thus a given credit spread. This is the theoretical spread we focus on the article.

The assessment of these risk factors can be repeated for very many companies to derive a credit portfolio risk profile.

Here next we follow a different route and rather than aiming at the valuation of portfolios default probabilities, we assess the dependence of market and theoretical spreads from the equity implied volatility expressed by the option market.
3.1 Market and theoretical spread dynamics

We develop a case study for the European telecom sector, focusing on the two market leaders, namely Deutsche Telekom and France Telecom. While limiting the analysis to those two companies, the collected evidences have been tested for a larger sample including companies in the Automotive and Utility sectors, as shown in Appendix.

Table 3.1.1 reports the correlation coefficients between actual and theoretical daily spreads within each sector. The sample period is Jan.2002-Dec.2003 and we consider daily observations.

<table>
<thead>
<tr>
<th>theor / actual</th>
<th>FORD</th>
<th>GM</th>
<th>DT</th>
<th>FT</th>
<th>ENDESA</th>
<th>RWE</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORD</td>
<td>0.7051</td>
<td>0.6069</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM</td>
<td>0.6968</td>
<td>0.5926</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DT</td>
<td></td>
<td></td>
<td>0.8707</td>
<td>0.7966</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FT</td>
<td></td>
<td></td>
<td>0.7795</td>
<td>0.7703</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENDESA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9157</td>
<td>0.6893</td>
</tr>
<tr>
<td>RWE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2470</td>
<td>0.7092</td>
</tr>
</tbody>
</table>

Table 3.1.1 Correlation between actual and theoretical spread values

The figures in the table support the introduction of normalised credit spreads constructed by subtracting from actual values the sample means and dividing by the statistical standard deviation. As z-scores they measure the deviation from the mean of daily values per unit volatility. These transformations leave the correlations unaltered.

For each company we have collected the required data input from January 1st 2002 to December 22nd 2003, to compute the theoretical spreads from Merton’s model. The maturity of the companies’ outstanding debt is assumed to be 5Y, in order to generate spread information comparable with the 5y CDS quotes.

These theoretical spread approximations, jointly with the implied volatility, are used to test:

- The dependence of actual spreads on stock price volatility
- The correlation between theoretical and actual spreads
- The differences between companies belonging to the same sector of activity
- The possibility to derive a measure of mis-pricing and as a consequence
- Trading indications within and across sectors.

The following figure reports the dynamics of theoretical (left) and actual (right) 5Y spreads for the Telecom sector. Dots are used to highlight market quotes for hedges actually completed. Thin lines indicate instead theoretical spreads.

As previously pointed out, together with the figures in appendix the figure shows that the hierarchy – creditworthiness – across theoretical spreads is preserved compared with market quotes.

Theoretical spreads are generally lower than actual spreads. When the equity volatility increases, however, theoretical spreads overall show a much higher volatility than the spreads expressed by the CDS market.
3.2 Equity volatility and market spreads during 2002-2003

The role played by the equity volatility in explaining the probability of default in Merton’s model has been analysed in § 2. We intend now to assess the relevance of such influence. CDS spreads are driven in general by several factors: stock price volatility is recognised to play a relevant role, together with other variables, such as the credit rating and several accounting ratios.

The strength of the relationship between stock price volatility and actual CDS spreads can thus vary over time and may in theory become negligible under specific conditions: the evidence we collect is instead that stock price volatility is a fundamental driver of spread movements, both theoretical and actual.

Furthermore, the calculated movements in theoretical spreads provide a robust approximation of forthcoming market spreads variations.

The late summer and early autumn of year 2002 represent epochal changes of the Telecom market structure and spread tightening was widespread.

The last year, instead, has witnessed a stable decrease of credit spreads with substantial movements of funds from the still unstable equity market to the bond market and Telecom’s have been among the beneficiaries.

The case of France Telecom is remarkable: figure 3.2.1 shows the very strong relationship between the equity implied volatility and the theoretical spreads during periods of increased volatility and, outside these periods, a close approximation of actual market movements by theoretical spreads.
Given the start-of-the-period theoretical spreads, changes of implied volatility do provide an extremely accurate approximation of theoretical spread variations. These in turn drive normalized actual spreads.

Summarizing, the two figures show:
High correlation between volatility patterns for the two companies and corresponding
High correlation between volatility trends and theoretical spreads
A significant impact of volatility spikes on theoretical spread movements
The tendency of CDS spreads to move tightly around the corresponding normalized theoretical spreads
During the last 12 months the decrease of implied volatility observed for both companies has been paralleled by a stabilization of theoretical spreads around the mean and by a convergence of actual spreads between the two companies.

3.3 CDS-Implied volatility lead-lag correlations and Granger causality

In table 3.1.1 we have reported daily correlation coefficients between theoretical and actual spreads for each company and within sectors. The evidence of strong infra-sector correlation has also emerged.

In this section we focus on the relationship between implied equity volatility and CDS spreads in order to test the presence of a leading effect of the former on the latter and if so the evidence of a Granger causality effect \(^1\) (see [Gourieroux and Monfort, 1997]).

The presence of a leading effect from, equity volatility to CDS spreads has far reaching practical consequences: to assess such a possibility we extend the correlation analysis in order to estimate lead-lag correlations based on a \(-3, +3\) day interval lags.

Due to lack of statistics on Endesa and RWE, whose CDS market as mention is not sufficiently liquid yet, the analysis is limited to the Automotive and Telecom sectors.

Table 3.3.1 displays the correlations between daily changes of implied volatility and CDS spreads, for the four considered companies.

For \(t = 1,2,...,T-3\), we report in the first row the estimated correlation between volatility changes recorded on day \(t\) and CDS spread changes on day \(t+3\), in the second the correlation between the former on day \(t\) and the latter on day \(t+2\) and so on. For each company in bold the maximum positive probability is highlight.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
ImpVol,CDS & FORD & GM & DT \\
\hline
-3 & 0.0369 & -0.0735 & -0.0048 \\
-2 & 0.0716 & 0.0115 & 0.0247 \\
-1 & 0.0593 & 0.0617 & 0.0269 \\
0 & -0.0399 & 0.0268 & 0.1338 \\
1 & 0.0502 & -0.0534 & -0.0647 \\
2 & -0.0908 & 0.0468 & -0.0062 \\
3 & 0.0386 & 0.0434 & 0.0224 \\
\hline
\end{tabular}
\caption{Lead-lag correlation between daily variations of implied volatility and CDS spreads}
\end{table}

In two out of four cases, namely Ford and GM, the figures support the hypothesis of a moderate leading effect of implied vol on CDS spreads. For Telecoms changes of implied volatility recorded the previous, two or three days before do not provide conclusive evidence on current variations of CDS spreads.

In general no decisive indications on a leading effect of implied volatility on credit spreads come from those results.

\(^1\) Granger causality studies the causal dependence of one variable from another. Positive lead correlation tends typically to be related with the presence for Granger causality, but as shown in the section not necessarily.
In order to collect more evidence on a possible causality direction from implied volatility to CDS spreads, we have developed a Granger causality test on the two variables. For the general theory on causality and exogeneity we refer again to [Gourieroux and Monfort, 1997]. We have tested both a lag 1 and lag 2 model of causality for the four companies: lag 1 Granger causality implies a causal effect on current spreads coming from the previous day implied volatility. This hypothesis is rejected – the null hypothesis accepted – depending on the value of a likelihood ratio, constructed as follows:

- The numerator is defined by the sum of squared residuals for a complete linear regression model containing lagged implied volatilities – to the second order in case of lag 2 causality testing and to the first order for lag 1 causality – and
- The denominator is determined by the sum of squared residuals in a simple autoregressive model with no exogenous variables considered.

As shown in tables 3.3.2 in three out of four cases the null hypothesis – implied volatility does not Granger-cause CDS spreads – is rejected at the 95% confidence level. Quite remarkably the R square is equal to 1 or near 1 in all estimated models and the p-value is 0.

In the table: mod1 refers to the complete model and mod2 to the simple autoregressive model with no exogenous variables.

| LAG1 | const | y-1 | impvol-1 | STATS | GRANGER CAUSALITY | NULL HYPOTH
|------|-------|-----|---------|-------|-----------------|---------------
| FORD |       |     |         | R^2   | F   | p             | S1       | F(1,T-3)  |
| mod1 | -0.0008 | 0.9479 | 0.0047 | 1     | 6025.1 | 0 | 10.3384 | 0.001480409 | reject |
| mod2 | 0.0001 | 0.9949 | 1       | 11602 | 0     |  |  |          |       |
| GM   |       |     |         | R^2   | F   | p             | S1       | F(1,T-3)  |
| mod1 | 0     | 0.9635 | 0.0025 | 1     | 3096.7 | 0 | 6.1834 | 0.013570708 | reject |
| mod2 | 0.0001 | 0.9941 | 1       | 6060.6 | 0     |  |  |          |       |
| DT   |       |     |         | R^2   | F   | p             | S1       | F(1,T-3)  |
| mod1 | -0.0008 | 0.8726 | 0.0051 | 1     | 14624  | 0 | 24.34  | 1.50248E-06 | reject |
| mod2 | 0     | 0.9942 | 1       | 26703 | 0     |  |  |          |       |
| FT   |       |     |         | R^2   | F   | p             | S1       | F(1,T-3)  |
| mod1 | 0.0001 | 0.9938 | -0.0004 | 1 | 13399 | 0 | 0.2327 | 0.629965036 | accept |
| mod2 | 0.0001 | 0.9879 | 1       | 26883 | 0     |  |  |          |       |

| LAG2 | const | y-1 | y-2 | impvol-1 | STATS | GRANGER CAUSALITY | NULL HYPOTH
|------|-------|-----|-----|---------|-------|-----------------|---------------
| FORD |       |     |     |         | R^2   | F   | p             | S1       | Chi_sq(2)  |
| mod1 | -0.0008 | 1.029 | -0.085 | 0.0041 | 0.008 | 1 | 2998 | 0 | 10.5088 | 0.0052245 | reject |
| mod2 | 0.0001 | 1.0752 | -0.0813 | 1 | 5794 | 0 | 7.1476 | 0.0280491 | reject |
| GM   |       |     |     |         | R^2   | F   | p             | S1       | Chi_sq(2)  |
| mod1 | 0.0001 | 1.0296 | -0.0862 | 0.0035 | -0.011 | 1 | 1547 | 0 | 31.365 | 1.523E-07 | reject |
| mod2 | 0.0001 | 1.0815 | -0.0651 | 1 | 3029 | 0 | 4.6836 | 0.0961544 | accept |
| DT   |       |     |     |         | R^2   | F   | p             | S1       | Chi_sq(2)  |
| mod1 | -0.001 | 0.8297 | 0.0191 | 0.0038 | 0.0022 | 1 | 7448.8 | 0 | 4.6836 | 0.0961544 | accept |
| mod2 | 0     | 0.9659 | 0.0291 | 1 | 13303 | 0 |  |          |       |
| FT   |       |     |     |         | R^2   | F   | p             | S1       | Chi_sq(2)  |
| mod1 | 0.0002 | 0.6866 | 0.3063 | 0.0031 | -0.0041 | 1 | 7647.1 | 0 | 4.6836 | 0.0961544 | accept |
| mod2 | 0.0001 | 0.8278 | 0.3581 | 1 | 15131 | 0 |  |          |       |

Table 3.3.2 Granger causality between implied volatility and CDS spreads

The null hypothesis is rejected in the LAG1 model by the F(1,T-3) statistics – where 1 and T-3 (here equal to 240) are the degrees of freedom for the F distribution— in presence of a likelihood ratio S1 greater than 3.087 at the 5% confidence level. It is rejected in the LAG2 model at the same confidence interval by the Chi-square(2) statistics when S1 exceeds 5.91.
In both models there is a strong evidence of a causality effect for DT and Ford, still present for GM, while there is no evidence of causality for FT.

A final judgement on the stability of this causality effect requires a more extended statistical back testing for a longer data horizon possibly considering moving time windows. This is left to future research.

As a summary it is possible to conclude that at least during year 2003, implied volatility has played a crucial role in determining spread levels and variations. Other variables such as risk free rates, leverage and stock prices playing a less crucial role.

In conclusion of this section a set of general results that have proven true for all companies considered in the study can also be sketched. In particular:

- Movements of implied volatility (up-down) are positively correlated with movements (up-down) of theoretical and actual spreads and tend to have a leading effect on these,
- Changes of actual spreads, when delayed, can be accurately inferred from changes of theoretical spreads,
- Actual spreads tend to fluctuate around the more volatile theoretical spreads,
- The difference between normalised spreads, both theoretical and actual, shows a cyclical behaviour around zero.

From the first point we can infer that equity and fixed income investors rely on common information sets and that higher (lower) risk perception in the equity market is rapidly transferred into wider (lower) spreads in the bond market.

From the second and the third points we can conclude that in the normalised world the proposed theoretical model provides a good benchmark for actual spread movements (even if no fair pricing appears possible).

Finally the fourth point is exploited in the mis-pricing analysis developed below.

4. RELATIVE VALUE ANALYSIS

The results presented support the evidence of a strong correlation between implied volatility and theoretical spreads and thereafter between theoretical and market spreads. Based on our pricing approach, we can thus conclude that stock price volatility, an input to Merton option model, represents the fundamental state variable of the model.

In this set-up the difference between CDS and theoretical spreads should indicate a possible over/under valuation of market spreads relative to theoretical values. Furthermore, if this is true, such a difference should fluctuate around an equilibrium – long term value of 0.

Figure 4.1 shows the behaviour of that difference for Deutsche and France Tlc during the 2002-2003 period.
The figure, as anticipated by the previous statistical analysis, shows that Telecoms present strong infra-sector correlations and there is a common credit factor driving the spreads of the two corporations.

Due to the random evolution of the normalised spreads, we see that the mis-pricing measure does change rapidly with occasional outliers, with sign changes even over short time periods. In order to avoid this variability we identify through a 3rd degree polynomial a persistent trend of the spreads difference and use this approximation as measure of price discrepancy. We consider two applications of the price discrepancy measure:

- **Cheap-dear analysis** that provides indication on optimal times to enter and exit the market
- **Convergence trade analysis** that allows the definition of possible long-short positions to exploit possible spreads convergence.

### 4.1 Mis-pricing measure and cheap-dear analysis

If the implied volatility is low (in relative terms) and the mis-pricing measure positive, than we should expect a movement of CDS spreads towards the theoretical spreads. Conversely if volatility is high, the theoretical spreads should be high and we should expect an increase of market spreads. Consider DT as an example.
On both figures the zero equilibrium level (solid grey line) on the mispricing axis corresponds to the point where actual spreads are equal to theoretical values. The smooth black line defines the current tendency of the mis-pricing condition.

Plot A displays the dynamics, relative to these two conditions, of the implied volatility (right Y axis). During the first part of year 2002 and then again since April 2003, implied vol is relatively low and, accordingly, theoretical spreads are low. Since the beginning of year 2003 there has been a general trend towards lower volatility coefficients. In March 2003 the mispricing measure becomes negative highlighting an underlying negative gap between theoretical and actual spreads: the latter, given the volatility pattern, is now expected to converge towards the former.

A similar analysis holds for France Telecom.
From September 2002 CDS spreads have started decreasing very rapidly after a period of instability. The descent started at a stage in which the mis-pricing measure was already negative: it stayed slightly negative during the rest of the year and then in the current year. In this period theoretical spreads are down, CDS spreads are down and below the theoretical value in normalized terms and volatility is low. It is not possible to derive here a general rule: right now CDS spreads appear at the current volatility level, fairly mispriced and an increase can be expected during the next few weeks.
4.2 Trading rules

In § 4.1 and the appendix, we show that CDS market movements can to a certain extent, be accurately foreseen by considering jointly the current level of stock price volatility and the current difference between market and theoretical spreads.

We test now whether by introducing buy-sell indications in correspondence of deviations from the 0 level of the mis-pricing measure and still conditioning on the behaviour of the ratio between CDS spread and equity volatility, we can enter and leave the market at the right time.

We consider France Telecom and limit the analysis to the year 2002. We assume that the investment decision relies on a 6-month history of actual CDS spreads, mispricing indicator and CDS/volatility ratio. At the beginning of the year the investor has no position on Ford and has collected the above data history.

The average CDS/volatility ratio, in green, has been computed from the original series through a linear fit. The mispricing indicator in black (relative to the red 0 line: a positive indicator indicates actual > theoretical normalised spreads) has instead been derived through a 3rd degree polynomial fit of the original series. These are compared with actual CDS spreads (left axis).

Figure 4.2.1 includes two plots: the first related to the market situation as at June 2002; the second as at mid August 2003.

Figure 4.2.1
Buy – Sell indications for FT credits, year 2003

The first plot shows the market evolution over the 6 months preceding June: over the last 4 weeks CDS spreads increased by more than 250bp diverging significantly from the average CDS/volatility ratio and driving up the mispricing indicator. The BUY signal is thus generated.

During the following months CDS spreads and stock market volatility start to decline, the latter faster than the former: as a result we observe a decrease of theoretical spreads and a convergence towards 0 of the mispricing indicator.
At the beginning of July the indicator becomes negative (actual spreads < theoretical spreads) and actual spreads stabilise: a SELL signal is now generated and a spread difference of 180 bps locked in.

### 4.3 Infra-sector relative values

We consider in this final section the potential of infra-sector relative value analysis based on essentially two sources of information: the differences between normalised theoretical spreads and between CDS market spreads for two different companies. The test is canonical: we consider whether significant differences between theoretical spreads drive the movements of market spreads for pair of companies.

Let’s consider Deutsche and France telecom: France Telecom being originally perceived by the market to be riskier than Deutsche Telekom during year 2002.

As before, the theoretical spread difference remains around 0 and since October 2002 moves remarkably in line with market spread differentials. In March 2003 the two credits are priced the same and over the summer the small difference in theoretical spreads converges to 0 as well.

A convergence trade [long France Tlc, short Deutsche Tlk] might have been constructed with profit generated during the period in which France telecom recovered.

### 5. CONCLUSIONS

In this report we have analysed in detail the interaction between stock price volatility, CDS market spreads and theoretical credit spreads for six representative companies in the automotive, telecom and utility sectors.
The stock price volatility implied out in the option market turns out to be a major driver of spread movements, both theoretical (not surprisingly, given the adopted model) and actual. The strong correlation between theoretical and actual spreads has been used to derive a risk-adjusted mispricing measure upon which market trading can be based.

The Merton model has been shown to provide a good benchmark model, despite of its limited ability to capture spreads’ fair values. We have pointed out that this generally recognised drawback does depend on the limited impact on theoretical spreads of changes in the liability structure. The model tends to undervalue the impact of changes in the leverage on the company default probability. This is instead a key variable in explaining market spread differences.

Observe however that the theoretical model has been shown to provide a coherent and robust reference model in absence of actively traded debt.

A major result of the study we have conducted can be found in the identification and the measurement of the key role of stock market volatility to define the risk source of credits and thus, the opportunity to rely on this input variable to assess forthcoming market movements. This result has been substantiated with a preliminary study on the presence of a causality effect from volatility patterns and CDS spreads: both in our setting estimated in the derivative market.

Acknowledgements
The results presented in this study come from a project run at UniCredit Banca Mobiliare Credit desk and the data have all being provided by the bank. The article has a companion similar publication published as UBM research by the bank and included in the bibliography. I acknowledge the fruitful collaboration with Daniele Palumbo, David Keeble and Giorgio Frascella from UBM on this project.
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APPENDIX. STATISTICAL ANALYSIS FOR AUTOMOTIVES AND UTILITIES

Here below, we report without comment, the results collected for Automotives and Utilities on the analysis described above for the Telecom sector. We refer to the article for comments on the displayed dynamics.

Figure A.1
Theoretical (left) and actual (right) 5Y spreads
Automotive and Utility sectors – 2002-2003
Figure A.2
Ford and GM
theoretical and actual spreads (left axis) versus equity implied volatility (right axis)

Figure A.3
Endesa and RWE
theoretical and actual spreads (left axis) versus equity implied volatility (right axis)
Figure A.4
Actual minus theoretical normalised spreads for the Automotive and Utility sectors

Ford (diamonds grey) & GM (squared black)
[Actual - Theoretical] 5Y Normalised Spreads

Endesa (black) & RWE (grey) [Actual - Theoretical] 5Y Spreads
Figure A.5
Mispricing measure vs implied volatility and CDS spreads. Jan 2002-Dec 2003
CDS and equity volatility: theoretical modelling and ... Giorgio CONSIGLI

RWE
Implied volatility vs mispricing measure

Jan-02 Apr-02 Jul-02 Sep-02 Jan-03 Apr-03 Jul-03 Oct-03

Jan-02 Apr-02 Jul-02 Oct-02 Jan-03 Apr-03 Jul-03 Oct-03

RWE
5Y CDS vs mispricing measure

actual > theor

actual < theor

Jan-02 Apr-02 Jul-02 Sep-02 Jan-03 Apr-03 Jul-03 Oct-03

actual > theor

actual < theor

Jan-02 Apr-02 Jul-02 Oct-02 Jan-03 Apr-03 Jul-03 Oct-03

Jan-02 Apr-02 Jul-02 Oct-02 Jan-03 Apr-03 Jul-03 Oct-03