Chapter 6: Price Discrimination: Nonlinear Pricing

Introduction

• Annual subscriptions generally cost less in total than one-off purchases
• Buying in bulk usually offers a price discount
  – these are price discrimination reflecting quantity discounts
  – prices are nonlinear, with the unit price dependent upon the quantity bought
  – allows pricing nearer to willingness to pay
  – so should be more profitable than third-degree price discrimination
• How to design such pricing schemes?
  – depends upon the information available to the seller about buyers
  – distinguish first-degree (personalized) and second-degree (menu) pricing
First-degree price discrimination 1

- Monopolist can charge maximum price that each consumer is willing to pay
- Extracts all consumer surplus
- Since profit is now total surplus, find that first-degree price discrimination is efficient

First-degree price discrimination 2

- Suppose that you own five antique cars
- Market research indicates that there are collectors of different types
  - keenest is willing to pay $10,000 for a car, second keenest $8,000, third keenest $6,000, fourth keenest $4,000, fifth keenest $2,000
  - sell the first car at $10,000
  - sell the second car at $8,000
  - sell the third car to at $6,000 and so on
  - total revenue $30,000
- Contrast with linear pricing: all cars sold at the same price
  - set a price of $6,000
  - sell three cars
  - total revenue $18,000
First-degree price discrimination 3

• First-degree price discrimination is highly profitable but requires
  – detailed information
  – ability to avoid arbitrage
• Leads to the efficient choice of output: since price equals marginal revenue and MR = MC
  – no value-creating exchanges are missed

First-degree price discrimination 4

• The information requirements appear to be insurmountable
  – but not in particular cases
    • tax accountants, doctors, students applying to private universities
• No arbitrage is less restrictive but potentially a problem
• But there are pricing schemes that will achieve the same outcome
  – non-linear prices
    – two-part pricing as a particular example of non-linear prices
      • charge a quantity-independent fee (membership?) plus a per unit usage charge
    – block pricing is another
      • bundle total charge and quantity in a package
Two-part pricing 1

- Jazz club serves two types of customer
  - Old: demand for entry plus \( Q_o \) drinks is \( P = V_o - Q_o \)
  - Young: demand for entry plus \( Q_y \) drinks is \( P = V_y - Q_y \)
  - Equal numbers of each type
  - Assume that \( V_o > V_y \): Old are willing to pay more than Young
  - Cost of operating the jazz club \( C(Q) = F + cQ \)
- Demand and costs are all in daily units

Two-part pricing 2

- Suppose that the jazz club owner applies “traditional” linear pricing: free entry and a set price for drinks
  - aggregate demand is \( Q = Q_o + Q_y = (V_o + V_y) - 2P \)
  - invert to give: \( P = (V_o + V_y)/2 - Q/2 \)
  - MR is then \( MR = (V_o + V_y)/2 - Q \)
  - equate MR and MC, where \( MC = c \) and solve for \( Q \) to give
    \( Q_U = (V_o + V_y)/2 - c \)
  - substitute into aggregate demand to give the equilibrium price
    \( P_U = (V_o + V_y)/4 + c/2 \)
  - each Old consumer buys \( Q_o = (3V_o - V_y)/4 - c/2 \) drinks
  - each Young consumer buys \( Q_y = (3V_y - V_o)/4 - c/2 \) drinks
  - profit from each pair of Old and Young is \( \pi_U = (V_o + V_y - 2c)^2 \)
Two part pricing 3

This example can be illustrated as follows:

Linear pricing leaves each type of consumer with consumer surplus.

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Two part pricing 4

• Jazz club owner can do better than this
• Consumer surplus at the uniform linear price is:
  – Old: \( CS_o = (V_o - P) \cdot Q_o / 2 = (Q_o)^2 / 2 \)
  – Young: \( CS_y = (V_y - P) \cdot Q_y / 2 = (Q_y)^2 / 2 \)
• So charge an entry fee (just less than):
  – \( E_o = CS_o \) to each Old customer and \( E_y = CS_y \) to each Young customer
    • check IDs to implement this policy
    – each type will still be willing to frequent the club and buy the equilibrium number of drinks
• So this increases profit by \( E_o \) for each Old and \( E_y \) for each Young customer
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Two-part pricing 5

- The jazz club can do even better
  - reduce the price per drink
  - this increases consumer surplus
  - but the additional consumer surplus can be extracted through a higher entry fee
- Consider the best that the jazz club owner can do with respect to each type of consumer

Two-Part Pricing

Set the unit price equal to marginal cost
This gives consumer surplus of \((V_i - c)^2/2\)
Set the entry charge to \((V_i - c)^2/2\)

Profit from each pair of Old and Young is now
\[
\pi_d = \frac{(V_o - c)^2 + (V_y - c)^2}{2}
\]
Block pricing

- There is another pricing method that the club owner can apply
  - offer a package of “Entry plus X drinks for $Y”
- To maximize profit apply two rules
  - set the quantity offered to each consumer type equal to the amount that type would buy at price equal to marginal cost
  - set the total charge for each consumer type to the total willingness to pay for the relevant quantity
- Return to the example:

\[
WTP_o = (V_o - c)^2/2 + (V_o - c)c = (V_o^2 - c^2)/2
\]
\[
WTP_y = (V_y - c)^2/2 + (V_y - c)c = (V_y^2 - c^2)/2
\]
Block pricing 3

- How to implement this policy?
  - card at the door
  - give customers the requisite number of tokens that are exchanged for drinks

A final comment

- One final point
  - average price that is paid by an Old customer = \( \frac{(V_o^2 - c^2)}{2(V_o - c)} \)
    = \( \frac{(V_o + c)}{2} \)
  - average price paid by a Young customer = \( \frac{(V_y^2 - c^2)}{2(V_o - c)} \) = \( \frac{(V_y + c)}{2} \)
  - identical to the third-degree price discrimination (linear) prices
  - but the profit outcome is much better with first-degree price discrimination. Why?
    - consumer equates MC of last unit bought with marginal benefit
    - with linear pricing MC = AC (= average price)
    - with first-degree price discrimination MC of last unit bought is less than AC (= average price) so more is bought
Second-degree price discrimination

- What if the seller cannot distinguish between buyers?
  - perhaps they differ in income (unobservable)
- Then the type of price discrimination just discussed is impossible
- High-income buyer will pretend to be a low-income buyer
  - to avoid the high entry price
  - to pay the smaller total charge
- Take a specific example
  - \( P_h = 16 - Q_h \)
  - \( P_l = 12 - Q_l \)
  - \( MC = 4 \)

Second-degree price discrimination 2

- First-degree price discrimination requires:
  - High Income: entry fee $72 and $4 per drink or entry plus 12 drinks for a total charge of $120
  - Low Income: entry fee $32 and $4 per drink or entry plus 8 drinks for total charge of $64
- This will not work
  - high income types get no consumer surplus from the package designed for them but get consumer surplus from the other package
  - so they will pretend to be low income even if this limits the number of drinks they can buy
- Need to design a “menu” of offerings targeted at the two types
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Second-degree price discrimination 3

- The seller has to compromise
- Design a pricing scheme that makes buyers
  - reveal their true types
  - self-select the quantity/price package designed for them
- Essence of second-degree price discrimination
  - It is “like” first-degree price discrimination
    - the seller knows that there are buyers of different types
    - but the seller is not able to identify the different types
- A two-part tariff is ineffective
  - allows deception by buyers
- Use quantity discounting

Second degree price discrimination 4

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Second-degree price discrimination 5

High-Income

Low-Income

Second-degree price discrimination 6

- Will the monopolist always want to supply both types of consumer?
- There are cases where it is better to supply only high-demand types
  - high-class restaurants
  - golf and country clubs
- Take our example again
  - suppose that there are \( N_l \) low-income consumers
  - and \( N_h \) high-income consumers
Second-degree price discrimination 7

- Suppose both types of consumer are served
  - two packages are offered ($57.50, 7) aimed at low-income and ($92, 12) aimed at high-income
  - profit is $31.50xN_l + $44xN_h

- Now suppose only high-income consumers are served
  - then a ($120, 12) package can be offered
  - profit is $72xN_h

- Is it profitable to serve both types?
  - Only if $31.50xN_l + $44xN_h > $72xN_h \Rightarrow 31.50N_l > 28N_h$

  This requires that \[ \frac{N_h}{N_l} < \frac{31.50}{28} = 1.125 \]

  There should not be “too high” a proportion of high-demand consumers

Second-degree price discrimination 8

- Characteristics of second-degree price discrimination
  - extract all consumer surplus from the lowest-demand group
  - leave some consumer surplus for other groups
    - the incentive compatibility constraint
    - offer less than the socially efficient quantity to all groups other than the highest-demand group
    - offer quantity-discounting

- Second-degree price discrimination converts consumer surplus into profit less effectively than first-degree

- Some consumer surplus is left “on the table” in order to induce high-demand groups to buy large quantities
Non-linear pricing and welfare

- Non-linear price discrimination raises profit
- Does it increase social welfare?
  - suppose that inverse demand of consumer group $i$ is $P = P_i(Q)$
  - marginal cost is constant at $MC - c$
  - suppose quantity offered to consumer group $i$ is $Q_i$
  - total surplus - consumer surplus plus profit - is the area between the inverse demand and marginal cost up to quantity $Q_i$.

Non-linear pricing and welfare 2

- Pricing policy affects
  - distribution of surplus
  - output of the firm
- First is welfare neutral
- Second affects welfare
- Does it increase social welfare?
- Price discrimination increases social welfare of group $i$ if it increases quantity supplied to group $i$. 
Non-linear pricing and welfare 2

- First-degree price discrimination always increases social welfare
  - extracts all consumer surplus
  - but generates socially optimal output
  - output to group $i$ is $Q_i(c)$
  - this exceeds output with uniform (non-discriminatory) pricing

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Non-linear pricing and welfare 3

- Menu pricing is less straightforward
  - suppose that there are two markets
    - low demand
    - high demand
- Uniform price is $P_U$
- Menu pricing gives quantities $Q_{1s}, Q_{2s}$
- Welfare loss is greater than $L$
- Welfare gain is less than $G$
Non-linear pricing and welfare 4

- It follows that
  \[\Delta W \leq G - L\]
  
  \[= (P_U - MC)\Delta Q_1 + (P_U - MC)\Delta Q_2\]
  
  \[= (P_U - MC)(\Delta Q_1 + \Delta Q_2)\]

- A necessary condition for second-degree price discrimination to increase social welfare is that it increases total output.
- "Like" third-degree price discrimination.
- But second-degree price discrimination is more likely to increase output.

The incentive compatibility constraint

- **Any offer made to high demand consumers must offer them as much consumer surplus as they would get from an offer designed for low-demand consumers.**
- This is a common phenomenon:
  - performance bonuses must encourage effort
  - insurance policies need large deductibles to deter cheating
  - piece rates in factories have to be accompanied by strict quality inspection
  - encouragement to buy in bulk must offer a price discount