Chapter 7: Product Variety and Quality under Monopoly

Introduction

- Most firms sell more than one product
- Products are differentiated in different ways
  - horizontally
    - goods of similar quality targeted at consumers of different types
      - how is variety determined?
      - is there too much variety
  - vertically
    - consumers agree on quality
    - differ on willingness to pay for quality
      - how is quality of goods being offered determined?
Horizontal product differentiation

- Suppose that consumers differ in their tastes
  - firm has to decide how best to serve different types of consumer
  - offer products with different characteristics but similar qualities
- This is horizontal product differentiation
  - firm designs products that appeal to different types of consumer
  - products are of (roughly) similar quality
- Questions:
  - how many products?
  - of what type?
  - how do we model this problem?

A spatial approach to product variety

- The spatial model (Hotelling) is useful to consider
  - pricing
  - design
  - variety
- Has a much richer application as a model of product differentiation
  - “location” can be thought of in
    - space (geography)
    - time (departure times of planes, buses, trains)
    - product characteristics (design and variety)
  - consumers prefer products that are “close” to their preferred types
    in space, or time or characteristics
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A Spatial approach to product variety 2

- Assume $N$ consumers living equally spaced along Main Street – 1 mile long.
- Monopolist must decide how best to supply these consumers
- Consumers buy exactly one unit provided that price plus transport costs is less than $V$.
- Consumers incur there-and-back transport costs of $t$ per mile
- The monopolist operates one shop
  - reasonable to expect that this is located at the center of Main Street
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The spatial model

Suppose the monopolist sets a price of $p_1$.

\[ z = 0 \]

\[ z = 1 \]

Shop 1

Price $p_1 + tx$

Price $p_1 + tx$

\[ p_1 + tx = V \], so $x = (V - p_1)/t$

What determines $x_1$?

All consumers within distance $x_1$ to the left and right of the shop will buy the product.

The spatial model 2

Suppose the firm reduces the price to $p_2$.

Then all consumers within distance $x_2$ of the shop will buy from the firm.

\[ z = 0 \]

\[ z = 1 \]

Shop 1

Price $p_1 + tx$

Price $p_1 + tx$

Then all consumers within distance $x_2$ of the shop will buy from the firm.

Suppose the firm reduces the price to $p_2$?
The spatial model 3

- Suppose that all consumers are to be served at price $p$.
  - The highest price is that charged to the consumers at the ends of the market
  - Their transport costs are $t/2$ : since they travel $\frac{1}{2}$ mile to the shop
  - So they pay $p + t/2$ which must be no greater than $V$.
  - So $p = V - t/2$.
- Suppose that marginal costs are $c$ per unit.
- Suppose also that a shop has set-up costs of $F$.
- Then profit is $\pi(N, 1) = N(V - t/2 - c) - F$.

Monopoly pricing in the spatial model

- What if there are two shops?
- The monopolist will coordinate prices at the two shops
- With identical costs and symmetric locations, these prices will be equal: $p_1 = p_2 = p$
  - Where should they be located?
  - What is the optimal price $p^*$?
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Location with two shops

Suppose that the entire market is to be served

If there are two shops they will be located symmetrically a distance d from the end-points of the market.

Now suppose that \( d < 1/4 \)

The shops should be moved inwards.

Delivered price to consumers at the market center equals their reservation price.

What determines \( p(d) \)?

Start with a low price at each shop.

Now raise the price at each shop.

Now suppose that \( d > 1/4 \)

The shops should be moved outwards.

Delivered price to consumers at the end-points equals their reservation price.

What determines \( p(d) \)?

Start with a low price at each shop.

Now raise the price at each shop.

The maximum price the firm can charge is now determined by the consumers at the end-points of the market.
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Location with two shops

It follows that shop 1 should be located at 1/4 and shop 2 at 3/4.

Price at each shop is then $p^* = V - t/4$.

Profit at each shop is given by the shaded area.

Profit is now $\pi(N, 2) = N(V - t/4 - c) - 2F$.

What if there are three shops?

By the same argument they should be located at 1/6, 1/2 and 5/6.

Price at each shop is now $V - t/6$.

Profit is now $\pi(N, 3) = N(V - t/6 - c) - 3F$.
Optimal number of shops

- A consistent pattern is emerging.
- Assume that there are \( n \) shops.
- They will be symmetrically located distance \( 1/n \) apart.
- We have already considered \( n = 2 \) and \( n = 3 \).
- When \( n = 2 \) we have \( p(N, 2) = V - t/4 \)
- When \( n = 3 \) we have \( p(N, 3) = V - t/6 \)
- It follows that \( p(N, n) = V - t/2n \)
- Aggregate profit is then \( \pi(N, n) = N(V - t/2n - c) - nF \)

Optimal number of shops 2

Profit from \( n \) shops is \( \pi(N, n) = (V - t/2n - c)N - nF \)
and the profit from having \( n + 1 \) shops is:
\[
\pi^*(N, n+1) = (V - t/(2(n + 1) - c)N - (n + 1)F
\]
Adding the \((n + 1)\)th shop is profitable if
\[
\pi(N, n+1) - \pi(N, n) > 0
\]
This requires
\[
tN/2n - tN/2(n + 1) > F
\]
which requires that \( n(n + 1) < tN/2F \).
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An example

Suppose that $F = $50,000, $N = 5$ million and $t = $1. Then $tN/2F = 50$

For an additional shop to be profitable we need $n(n + 1) < 50$.
This is true for $n \leq 6$
There should be no more than seven shops in this case: if $n = 6$ then adding one more shop is profitable.
But if $n = 7$ then adding another shop is unprofitable.

Some intuition

- What does the condition on $n$ tell us?
- Simply, we should expect to find greater product variety when:
  - there are many consumers.
  - set-up costs of increasing product variety are low.
  - consumers have strong preferences over product characteristics and differ in these
    - consumers are unwilling to buy a product if it is not “very close” to their most preferred product
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How much of the market to supply

• Should the whole market be served?
  – Suppose not. Then each shop has a local monopoly
  – Each shop sells to consumers within distance \( r \)
  – How is \( r \) determined?
    • it must be that \( p + tr = V \) so \( r = (V - p)/t \)
    • so total demand is \( 2N(V - p)/t \)
    • profit to each shop is then \( \pi = 2N(p - c)(V - p)/t - F \)
    • differentiate with respect to \( p \) and set to zero:
      • \( d\pi/dp = 2N(V - 2p + c)/t = 0 \)
      • So the optimal price at each shop is \( p^* = (V + c)/2 \)
    – Only part of the market should be served if \( p(N,n) < p^* \)
    – This implies that \( V < c + t/n \).

Partial market supply

• If \( c + t/n > V \) supply only part of the market and set price \( p^* = (V + c)/2 \)
• If \( c + t/n < V \) supply the whole market and set price \( p(N,n) = V - t/2n \)
• Supply only part of the market:
  – if the consumer reservation price is low relative to marginal production costs and transport costs
  – if there are very few outlets
Social optimum

What number of shops maximizes total surplus?

Total surplus is consumer surplus plus profit

Consumer surplus is total willingness to pay minus total revenue

Profit is total revenue minus total cost

Total surplus is therefore $N.V - Total Cost$

So what is Total Cost?

Assume that there are $n$ shops

Consider shop $i$

Total cost is total transport cost plus set-up costs

Transport cost for each shop is the area of these two triangles multiplied by consumer density

This area is $t/4n^2$
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Social optimum 3
Total cost with \( n \) shops is, therefore:
\[
C(N,n) = n(t/4n^2)N + n.F = tN/4n + n.F
\]
Total cost with \( n + 1 \) shops is:
\[
C(N,n+1) = tN/4(n+1)^2 + (n+1).F
\]
Adding another shop is socially efficient if \( C(N,n + 1) < C(N,n) \)
This requires that \( tN/4n - tN/4(n+1) > F \)
which implies that \( n(n + 1) < tN/4F \)

There should be five shops: with \( n = 4 \) adding another shop is efficient

The monopolist operates too many shops and, more generally, provides too much product variety

Product variety and price discrimination

- Suppose that the monopolist delivers the product.
  - then it is possible to price discriminate
- What pricing policy to adopt?
  - charge every consumer his reservation price \( V \)
  - the firm pays the transport costs
  - this is uniform delivered pricing
  - it is discriminatory because price does not reflect costs
- Should every consumer be supplied?
  - suppose that there are \( n \) shops evenly spaced on Main Street
  - cost to the most distant consumer is \( c + t/2n \)
  - supply this consumer so long as \( V \) (revenue) > \( c + t/2n \)
  - This is a weaker condition than without price discrimination.
  - Price discrimination allows more consumers to be served.
Product variety and price discrimination 2

• How many shops should the monopolist operate now?

Suppose that the monopolist has \( n \) shops and is supplying the entire market.

Total revenue minus production costs is \( N.V - N.c \)

Total transport costs plus set-up costs is \( C(N, n) = \frac{tN}{4n} + n.F \)

So profit is \( \pi(N, n) = N.V - N.c - C(N, n) \)

But then maximizing profit means minimizing \( C(N, n) \)

*The discriminating monopolist operates the socially optimal number of shops.*

Monopoly and product quality

• Firms can, and do, produce goods of different qualities
• Quality then is an important strategic variable
• The choice of product quality determined by its ability to generate profit; attitude of consumers to quality
• Consider a monopolist producing a single good
  – what quality should it have?
  – determined by consumer attitudes to quality
    • prefer high to low quality
    • willing to pay more for high quality
    • *but* this requires that the consumer recognizes quality
    • *also* some are willing to pay more than others for quality
Demand and quality

- We might think of individual demand as being of the form
  - \( Q_i = 1 \) if \( P_i \leq R_i(Z) \) and = 0 otherwise for each consumer \( i \)
  - Each consumer buys exactly one unit so long as price is less than her reservation price
  - The reservation price is affected by product quality \( Z \)
- Assume that consumers vary in their reservation prices
- Then aggregate demand is of the form \( P = P(Q, Z) \)
- An increase in product quality increases demand

Demand and quality 2

Begin with a particular demand curve for a good of quality \( Z_1 \)

Then an increase in product quality from \( Z_1 \) to \( Z_2 \) rotates the demand curve around the quantity axis as follows

Quantity \( Q_1 \) can now be sold for the higher price \( P_2 \)

These are the inframarginal consumers

This is the marginal consumer
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Demand and quality 3

Suppose instead that an increase in quality increases the willingness to pay of marginal consumers more than that of the inframarginal consumers.

Then an increase in product quality from $Z_1$ to $Z_2$ rotates the demand curve around the price axis as follows.

Once again quantity $Q_1$ can now be sold for a higher price $P_2$.

Demand and quality 4

- The monopolist must choose both
  - price (or quantity)
  - quality
- Two profit-maximizing rules
  - marginal revenue equals marginal cost on the last unit sold for a given quality
  - marginal revenue from increased quality equals marginal cost of increased quality for a given quantity
- This can be illustrated with a simple example:

$$P = Z(\theta - Q)$$

where $Z$ is an index of quality.
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Demand and quality 5

\[ P = Z(\theta - Q) \]

Assume that marginal cost of output is zero: \( MC(Q) = 0 \)

Cost of quality is \( D(Z) = \alpha Z^2 \)

Marginal cost of quality = \( dD(Z)/d(Z) = 2\alpha Z \)

This means that quality is costly and becomes increasingly costly.

The firm’s profit is:

\[ \pi(Q, Z) = P \cdot Q - D(Z) = Z(\theta - Q)Q - \alpha Z^2 \]

The firm chooses \( Q \) and \( Z \) to maximize profit.

Take the choice of quantity first: this is easiest.

Marginal revenue = \( MR = Z\theta - 2ZQ \)

\[ MR = MC \Rightarrow Z\theta - 2ZQ = 0 \Rightarrow Q^* = \theta/2 \]

\[ \therefore P^* = Z\theta/2 \]

Demand and quality 6

Total revenue = \( P^*Q^* = (Z\theta/2)\cdot(\theta/2) = Z\theta^2/4 \)

So marginal revenue from increased quality is \( MR(Z) = \theta^2/4 \)

Marginal cost of quality is \( MC(Z) = 2\alpha Z \)

Equating \( MR(Z) = MC(Z) \) then gives \( Z^* = \theta^2/8\alpha \)

Does the monopolist produce too high or too low quality?
Demand and quality: multiple products

- What if the firm chooses to offer more than one product?
  - what qualities should be offered?
  - how should they be priced?
- Determined by costs and consumer demand
- An example:
  - two types of consumer
  - each buys exactly one unit provided that consumer surplus is nonnegative
  - if there is a choice, buy the product offering the larger consumer surplus
  - types of consumer distinguished by willingness to pay for quality
- This is vertical product differentiation

Vertical differentiation

- Indirect utility to a consumer of type $i$ from consuming a product of quality $z$ at price $p$ is $V_i = \theta_i(z - z_i) - p$
  - where $\theta_i$ measures willingness to pay for quality;
  - $z_i$ is the lower bound on quality below which consumer type $i$ will not buy
  - assume $\theta_1 > \theta_2$: type 1 consumers value quality more than type 2
  - assume $z_1 > z_2 = 0$: type 1 consumers only buy if quality is greater than $z_1$:
    - never fly in coach
    - never shop in Wal-Mart
    - only eat in “good” restaurants
  - type 2 consumers will buy any quality so long as consumer surplus is nonnegative
Vertical differentiation 2

- Firm cannot distinguish consumer types
- Must implement a strategy that causes consumers to *self-select*
  - persuade type 1 consumers to buy a high quality product $z_1$ at a high price
  - and type 2 consumers to buy a low quality product $z_2$ at a lower price, which equals their maximum willingness to pay
- Firm can produce any product in the range $[z_2, z_1]$
- MC = 0 for either quality type

Vertical differentiation 3

Suppose that the firm offers two products with qualities $z_1 > z_2$

For type 2 consumers charge maximum willingness to pay for the low quality product: $p_2 = \theta_2 z_2$

Now consider type 1 consumers: firm faces an *incentive compatibility constraint*

\[
\theta_1 (z_1 - z_1) - p_1 \geq \theta_1 (z_2 - z_1) - p_2
\]

\[
\theta_1 (z_1 - z_1) - p_1 \geq 0
\]

These imply that $p_1 \leq \theta_1 z_1 - (\theta_1 - \theta_2) z_2$

There is an upper limit on the price that can be charged for the high quality good.
Vertical differentiation 4

- Take the equation $p_1 = \theta_1 z_1 - (\theta_1 - \theta_2)z_2$
  - this is increasing in quality valuations
  - increasing in the difference between $z_1$ and $z_2$
  - quality can be priced highly when it is valued highly
  - firm has an incentive to differentiate the two products’ qualities to
    soften competition between them
    - monopolist is competing with itself

- What about quality choice?
  - prices $p_1 = \theta_1 z_1 - (\theta_1 - \theta_2)z_2$; $p_2 = \theta_2 z_2$
  - check the incentive compatibility constraints
  - suppose that there are $N_1$ type 1 and $N_2$ type 2 consumers

Vertical differentiation 5

Profit is $\Pi = N_1 p_1 + N_2 p_2 = N_1 \theta_1 z_1 - (N_1 \theta_1 - (N_1 + N_2)\theta_2)z_2$

This is increasing in $z_1$ so set $z_1$ as high as possible: $z_1 = z$

For $z_2$ the decision is more complex

$(N_1 \theta_1 - (N_1 + N_2)\theta_2)$ may be positive or negative
Vertical differentiation 6

Case 1: Suppose that $(N_1 \theta_1 - (N_1 + N_2) \theta_2)$ is positive

Then $z_2$ should be set “low” but this is subject to a constraint

Recall that $p_1 = \theta_1 z_1 - (\theta_1 - \theta_2) z_2$. So reducing $z_2$ increases $p_1$

But we also require that $\theta_1 (z_1 - \bar{z}_1) - p_1 \geq 0$

Putting these together gives:

$$z_2 = \frac{\theta_1 z_1}{\theta_1 - \theta_2}$$

$$p_2 = \frac{\theta_2 \theta_1 z_1}{\theta_1 - \theta_2}$$

The equilibrium prices are then:

$$p_1 = \theta_1 (\bar{z} - \bar{z}_1)$$

Vertical differentiation 7

- Offer type 1 consumers the highest possible quality and charge their full willingness to pay
- Offer type 2 consumers as low a quality as is consistent with incentive compatibility constraints
- Charge type 2 consumers their maximum willingness to pay for this quality
  - maximum differentiation subject to incentive compatibility constraints
Vertical differentiation 8

Case 1: Now suppose that \((N_1 \theta_1 - (N_1 + N_2 \theta_2))\) is negative

Then \(z_2\) should be set as high as possible

The firm should supply only one product, of the highest possible quality

What does this require?

From the inequality offer only one product if: 
\[
\frac{N_1}{N_1 + N_2} < \frac{\theta_2}{\theta_1} < 1
\]

Offer only one product:
- if there are not “many” type 1 consumers
- if the difference in willingness to pay for quality is “small”

Should the firm price to sell to both types in this case? Yes!

Demand and quality A1

How does increased quality affect demand?
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Demand and quality A2

Price

Z_2\theta

Z_1\theta

P_2 = Z_2\theta/2

P_1 = Z_1\theta/2

θ/2
θ
Q^*

Quantity

Social surplus at quality Z_1 is this area minus quality costs

Social surplus at quality Z_1 is this area minus quality costs

The increase is total surplus is greater than the increase in profit.
The monopolist produces too little quality.

Demand and quality

Derivation of aggregate demand

Order consumers by their reservation prices

Aggregate individual demand horizontally

Price

1 2 3 4 5 6 7 8

Quantity
Location choice 1

\( d < 1/4 \)

We know that \( p(d) \) satisfies the following constraint:

\[ p(d) + t(1/2 - d) = V \]

This gives:

\[ p(d) = V - t/2 + t.d \]

\[ \therefore p(d) = V - t/2 + t.d \]

Aggregate profit is then:

\[ \pi(d) = (p(d) - c)N \]

\[ = (V - t/2 + t.d - c)N \]

This is increasing in \( d \) so if \( d < 1/4 \) then \( d \) should be increased.

Location choice 2

\( d > 1/4 \)

We now know that \( p(d) \) satisfies the following constraint:

\[ p(d) + t.d = V \]

This gives:

\[ p(d) = V - t.d \]

Aggregate profit is then:

\[ \pi(d) = (p(d) - c)N \]

\[ = (V - t.d - c)N \]

This is decreasing in \( d \) so if \( d > 1/4 \) then \( d \) should be decreased.