Valuation Accuracy and Infinity Horizon Forecast: Empirical Evidence from Europe

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Abstract

This paper focuses on the assumptions of infinite-horizon forecasting in the field of firm valuation. The estimate of long-run continuing values is based on the hypothesis that companies should have reached the steady state at the end of the period of explicit forecasts. It is argued that the equivalence between cash accounting and accrual accounting is the way of verifying the steady-state assumption, defined as the state when a firm earns exactly its cost of capital, i.e., what we would expect in pure-competition settings. From this definition, we derive that the “ideal” growth rate to use in steady state is equal to the reinvestment rate times Weighted Average Cost of Capital. To validate our approach, we collect a sample of 784 analyst valuations and compare how the implied target prices deviate from what the target prices would have been using the “ideal” steady-state growth rates. Using Logit and Cox regression models, we find that this deviation has predictive value over the probability that actual market price reaches the target price over the following 12-month period—the smaller the deviation the greater is the likelihood that the market price reaches the target price.

1. Introduction

To value firms, analysts have to forecast payoffs. If one were to forecast to infinity, one could forecast dividends, as these are the payoff to holding shares. However, even though going concerns are considered to continue indefinitely, practical analysis typically deals with finite forecast horizons, presumably for reasons of bounded rationality. The need for alternatives to dividends arises because forecasting dividends over a finite horizon is not very informative.

Valuation models like Discounted Cash Flow (DCF) models, earnings capitalization models, and Residual Income (RI) valuation models offer
alternative specifications for this issue. As is often stated, in general terms valuations are equivalent to infinite forecasting horizons (Feltham and Ohlson, 1995; Koller et al., 2005; Penman, 2007). And if alternative models equivalent for infinite-horizon forecasts are thus stated, they necessarily yield equivalent valuations (Francis et al., 2000; Courteau et al., 2001). On the other hand, the need for finite-horizon forecasting is the rationale behind considering alternative valuation models instead of the dividend discount model and the critical point relies on the definition of infinite-horizon forecasting. The choice between cash accounting and accrual accounting is therefore at the very heart of accounting research. The debate is ongoing. For instance, Penman advocates RI models, while Koller et al. (2005) recommend using the DCF declaring that “cash is king”. These authors also state that DCF is most widely used in practice, although RI is gaining popularity. This paper concentrates on the hypotheses adopted for such infinite-horizon forecasting.

From a two-stage DCF perspective, the focus is on the transition from the first period of explicit forecasts to the second stage of infinite horizon. Theoretically, the end of the first stage implies the end of any possible source of extra profitability and the beginning of a steady-state period. The beginning of the second stage indeed represents the end of a period of competitive advantage, also known as the Competitive Advantage Period (Mauboussin and Johnson, 1997). As a result, the dynamic of future cash flow growth in the second stage is expressed as a function of a unique steady-state growth rate. However, it is not the stable growth rate that drives value as much as what it is assumed about excess returns in perpetuity (Damodaran, 2006). As such, the equivalence between the DCF and the RI model represents an element of verification of the assumptions underlying the forecast of the continuing value.

The aim of this paper is to provide a means of measurement that indicates how accurate DCF company valuations are in relation to the long-term sustainability of the hypotheses implicit in the valuation. Specifically, the company is tested to see if it has reached the steady state at the end of the first period of individual forecasts. The verification criterion is based on the idea that it is not possible to hypothesize that the company can maintain a competitive advantage for an infinite period of time, and therefore a positive difference between profitability and cost of capital invested. The breaking of the convergence condition can lead to an over optimistic (or pessimistic) company valuation due to lack of coherence in the assumptions of steady growth.
This study proposes an index that tests the adoption of the steady-state assumptions when estimating the continuing value and it is the first to provide large-sample empirical evidence on the importance of steady-state modeling in cash flow valuation models. Such accuracy index reveals possible violations of the convergence conditions between return and cost of capital within the infinite-horizon forecasts. Its efficacy is validated using a sample of 784 equity reports of sell-side analysts published in the period 2003–2005 valuing listed European companies. The final sample is selected using software-assisted content analysis from an original sample of over 150,000 reports available online on Investext database. Using Logit and Cox regression models, it is found that valuations with a lower index (i.e., coherent with the assumption of long-term steady state) have a higher probability that their target price reaches the actual trade price within 12 months from the publication of the equity report. In other words, the definition of a long-term constant growth rate in accordance with steady-state hypothesis leads to better performing valuations.

This paper is organized as follows. The following section reviews the literature. Section 3 relates the DCF to the RI model and proposes the accuracy index. Section 4 describes the sample of equity reports. The empirical validation of the accuracy index is presented in section 5 using Logit and Cox regression models. Section 6 presents the conclusions.

2. Review of the Literature

Several authors compare value estimates using alternative equity valuation models with actual traded prices. The topics addressed in this literature typically examine and compare the valuation performance of the RI valuation model and its counterparts, the dividend discount model (DDM) and the DCF valuation model; accounting based measures of expected returns and accounting based multiplier models. Bernard (1995) focuses the RI model (Ohlson, 1995) and its inherent advantages over the dividend discounting approach to valuation. Penman and Sougiannis (1998) examine models that forecast dividends, cash flow, earnings, or book values, and also compare models that capitalize forecasted earnings rather than discount residual earnings. Francis et al. (2000) extend the comparison by using ex ante analysts’ forecasts (whereas Penman and Sougiannis used ex post averages). Numerous later studies (e.g., Berkman et al., 2000; Gilson et al., 2000) compare
different valuation methodologies and generally provide empirical support to valuation using cash accounting or accrual accounting.

Recently, a growing research interest has focused on the infinite-horizon forecasts. “This surely is worthwhile of empirical investigation” (Penman, 2001, p. 685) as there is no real consensus on the method of calculating the continuing value. In order to do this, Penman and Sougiannis (1998) investigate valuations with truncated forecasting horizons and experiment with different ad hoc growth rates in continuing values. Courteau et al. (2001) explicitly examine the consequences of using ad hoc continuing values. This paper contributes to this literature by investigating the steady-state assumptions for the infinite-horizon forecasts.

3. DCF, RI, and Steady-State Assumptions

3.1 DCF and RI Model

Like other firm valuation models, the DCF model proceeds in two periods. For each year in the explicit forecast period, there is an individual forecast of free cash flow. On the other hand, each year in the post-horizon period is represented by one single continuing value formula, being the steady-state value of the firm’s assets at the horizon. This second stage leads to the calculation of the continuing value of the company in which the dynamic of prospective cash flows is determined by using a steady-state growth rate only.

However, being based on cash accounting rather than accrual accounting, the DCF does not directly reflect the hypotheses of operating performance. Consequently, the assumptions upon which the valuation process is based are not immediately verifiable. This particularly applies to the sustainability of competitive advantage and the estimation of long-term parameters. In this context, an improvement in the “verifiability” of the valuation process is obtained by reconciling the DCF and RI models. RIs are defined as the difference between return and cost of capital (notation in Appendix E):

\[ RI_t = (ROIC_t - WACC) \cdot IC_{t-1} \] (1)

where ROIC is the expected rate of return on new investments given by the ratio between the Net Operating Profit After Tax and the Invested Capital \( ROIC_t = NOPAT_t / IC_{t-1} \).

In agreement with Feltham and Ohlson (1995), the variation of operating capital invested in the company is equal to the difference
between the net operating income and the cash flow available for the investors. This relation is known as “Operating Assets Relation” and relates the DCF model to the generation of economic value. The economic value of the company is therefore determined by the sum of the book value of its capital invested and the present value of the expected RIs (Feltham and Ohlson, 1995; proof in Appendix A):

\[ EV_t = IC_t + \sum_{i=1}^{\infty} \frac{RI_{t+i}}{(1 + WACC)^i} \]  

(2)

The DCF model and the RI model produce identical results in that they underline identical hypotheses on the future of the company under valuation (Koller et al., 2005; Penman, 2007). However, the equivalence of the two approaches in applications is only conserved if the analytical forecasts continue for infinity, while interruption of the forecast at a finite point and the consequent use of a continuing value is the source of disagreement between the results produced by the two methods (Penman, 2007). Nevertheless, the opportunity to trace the DCF model back to the RI model is a formal instrument that can be used to formulate criteria that in turn can be used to verify the conditions for calculating the continuing value. Numerous studies provide empirical support to the approach based on the definition of RIs (for instance, Penman and Sougiannis, 1998; Francis et al., 2000).

3.2 Steady-State Assumptions

The definition of a perpetual steady state is based on the hypothesis that at the end of the period of explicit forecasts, the company should have reached a steady state. Consequently, in confirming the transition to steady state, the end of the stage of individual forecasts coincides with the end of any source of extra profitability due to competition forces. The beginning of the final stage represents the end of a period of competitive advantage, defined as Competitive Advantage Period by Mauboussin and Johnson (1997). This transition occurs when the expected profitability from new investments stops being greater than the cost of capital. Therefore, from that moment the company is assumed to be in a steady state that continues to infinity (Koller et al., 2005).

As a result of making this assumption, the dynamic of future cash flow growth is expressed as a function of the unique steady-state growth rate \( g_2 \). Consequently, the estimation of \( g_2 \) is one of the most critical in that
the estimated Enterprise Value is very sensitive to this parameter. Therefore, the equivalence between the DCF and the RI model represents an element of verification of the assumptions of effective steady state. The verification criterion is based on the idea that it is not possible to hypothesize that the company can maintain a competitive advantage, and therefore a positive difference between profitability and cost of capital, for an infinite period of time. In fact, due to the effect of market forces it can be hypothesized that sources of extra profitability due to competitive advantage will sooner or later come to an end. Moreover, the hypotheses on which the forecasts of the continuing value are based should be such as to imply an asymptotic convergence of the profitability forecast with the cost of capital. Consequently, two aspects need to be considered. On the one hand, differentiating the contribution made to the operating profit of existing investments from that made by the new investments that the company will make during the period of steady state. On the other hand, relating the trend of the variables to the situation in which it is hypothesized that the company is in the final year of explicit forecasting. As a result of these requirements, the following conditions are introduced to identify the steady state (Koller et al., 2005):

(i) The incremental return on new invested capital is constant during the steady state and equal to that estimated for the terminal year $ROIC_T$.

(ii) The incremental return on new invested capital is constant, so the average return on invested capital varies in the second stage only as a result of new investments.

(iii) The reinvestment rate (defined as net investment over operating profits) is constant during the steady state and it is equal to the investment rate in the final year of explicit forecast $h_T$.

Since average Return On Invested Capital varies as a function of the new investments and the marginal profitability, the growth in cash flow ($g_2$) can be expressed as the product of marginal profitability ($ROIC_T$) and the coefficient of reinvestment for the terminal year ($h_T$). Furthermore, assuming a positive investment strategy, the incremental return on new invested capital represents company profit in the long run. Consequently, the marginal profitability represents a level of long-term profitability pursued by the company. The condition of asymptotic equivalence between return and cost of capital is satisfied if the perpetual growth rate of cash flows $g_2$ is defined by the following equation (proof in Appendix B)³:

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$g^*_2 = \frac{WACC \cdot h_T}{1 + g^*_2}$  \hspace{1cm} (3)

where $g^*_2$ is the “ideal” steady-state growth rate that ensures the exact asymptotic convergence between return and cost of capital.

The analysis of the equity reports shows how the convergence condition is not always adopted in the DCF valuation. So the hypotheses used in calculating the continuing value implicitly assume that a differential between profitability and cost of capital other than zero is maintained for an unlimited period. This, together with a constant increase in operating capital due to new investments, means a monotonous trend of growth (decline) in RIs in the steady state, if the differential is positive (negative). The breaking of the convergence condition can lead to an over optimistic (pessimistic) valuation due to lack of coherence in the assumptions of steady state. In conclusion, the divergence with the “ideal” case is reflected in a deviation of the target price (to an extent that depends on the leverage of the firm).

In order to identify the possible presence of overconfidence in the valuation procedure, the following variance index was defined:

$$\Delta TP\% = \frac{TP - TP^*}{TP}$$  \hspace{1cm} (4)

where $TP$ is the target price obtained using DCF applying the effective perpetual growth rate estimated in the equity report, whereas $TP^*$ is that which would result from applying the ideal $g^*_2$ rate.

This index therefore measures the impact on the target price of not observing the long-term convergence condition. If there is a positive difference between return and cost of capital implied in the assumptions in the calculation of the continuing value, $\Delta TP\%$ assumes a value greater than zero and is as large as the size of the variance from the ideal case. Vice versa, $\Delta TP\%$ assumes negative values if company profitability is asymptotically less than its cost.

4. The Sample of Equity Reports

To validate our variance index 784 equity reports were selected from the Investext database. In order to be selected, the equity report must (1) be published in the period from 2003 to 2005; (2) value a European company listed on the stock exchanges of one of the four largest European economies: London Stock Exchange (UK), Euronext (France, Belgium, the Netherlands, Portugal), Deutsche Börse (Germany) and
Borsa Italiana (Italy); (3) value an operating company; (4) use the DCF model and report explicitly detailed information regarding the stream of expected cash flows, the estimate of cost of capital, and all the hypothesis for the long-term.  

Most of the selected equity reports were on German or British companies (253 and 244 reports, respectively), the rest on Italian (118), French (96), Dutch (57), and Belgian (16) firms. 244 equity reports were published in 2003, 247 in 2004, and 293 in 2005.

Table 1 describes the sample of equity reports. The period covered by the first stage of explicit forecast is an average of 8.7 years. The average value of the steady-state growth rate \( g_2 \) is 1.77 per cent. As expected, a consistent percentage of the estimated equity value is on average attributable to the terminal value (63 per cent). The average value of the variance index \( \Delta TP\% \) is equal to 6.39 per cent. Surprisingly, a few valuations have a negative investment rate in the terminal year. In these cases, the estimated terminal cash flow \( (FCFF_T) \) exceeds the profit generated in that year, which means a strategy of disinvestment in the steady state.

Table 2 reports the descriptive statistics of the sample partitioned by recommendation. A three level scale of recommendations is used: Sell, Hold, and Buy. Most of the equity reports in the sample have a Buy recommendation (60 per cent), followed by Hold (30 per cent). The frequency with which Sell is recommended is lower (10 per cent), but still greater than that reported in other studies (see, for example, Asquith et al., 2005). The difference between the target price and the current stock price is also measured, scaled to the current stock price. As expected, it is found that the difference increases from Sell recommendations (with on average a target price of \(-10.4\) per cent relative to the current trade price), to Hold (6.0 per cent) and Buy (22.7 per cent).

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**Table 1. Sample description**

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>25th percentile</th>
<th>75th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T ) (years)</td>
<td>8.72</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>( g_2 ) (%)</td>
<td>1.77</td>
<td>1.00</td>
<td>2.50</td>
</tr>
<tr>
<td>( TV/EV ) (%)</td>
<td>63.42</td>
<td>54.75</td>
<td>71.82</td>
</tr>
<tr>
<td>( h_T ) (%)</td>
<td>12.10</td>
<td>13.25</td>
<td>21.52</td>
</tr>
<tr>
<td>( \Delta TP% ) (%)</td>
<td>6.39</td>
<td>0.20</td>
<td>13.95</td>
</tr>
</tbody>
</table>

\( T \) is the length of the explicit forecast period, \( g_2 \) is the steady-state growth rate, \( TV/EV \) the weight of the Terminal Value over the whole Enterprise Value, \( h_T \) investment rate for the terminal year, and \( \Delta TP\% \) is the variance index defined in the paper.
Moreover, target prices published in equity reports in support of the recommendation do not necessarily coincide with the target price obtained using the DCF model. Indeed, the analysts usually use different valuation methods and the published target price is the result of the estimates of these models. Two features were defined to investigate the effective use of the DCF in determining the final target price. Firstly, $D_{DCF}$ is the difference between the target price published in the report and that obtained using the DCF method. This measure provides information on how optimistic/pessimistic DCF valuations are compared with the target price published in the equity reports. On the other hand, $|D_{DCF}|$ expresses the difference in modulus, and quantifies the extent to which the final target price is different from the DCF target price, without taking the sign into consideration. A negative average $D_{DCF}$ is found, which suggests that the target prices obtained using the DCF are on average more optimistic than the published ones, both for the entire sample and for each individual group of recommendations. There is a tendency for the absolute difference $|D_{DCF}|$ to decrease the more favorable the recommendation is (the difference in the average values

Table 2. Descriptive statistics by recommendation (averages)

<table>
<thead>
<tr>
<th></th>
<th>Sell</th>
<th>Sell/Hold</th>
<th>Hold</th>
<th>Sell/Hold Buy</th>
<th>Sell/Hold Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Reports</td>
<td>80</td>
<td>234</td>
<td>470</td>
<td>784</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.2%)</td>
<td>(29.8%)</td>
<td>(59.9%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel A: Report characteristics

<table>
<thead>
<tr>
<th></th>
<th>Sell</th>
<th>Sell/Hold</th>
<th>Hold</th>
<th>Sell/Hold Buy</th>
<th>Sell/Hold Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(TP - P_0)/P_0$ (%)</td>
<td>$-10.40$</td>
<td>$10.07^{***}$</td>
<td>$6.01$</td>
<td>$12.34^{***}$</td>
<td>$22.74$</td>
</tr>
<tr>
<td>$\Delta DCF$ (%)</td>
<td>$-5.65$</td>
<td>$1.73^*$</td>
<td>$-1.86$</td>
<td>$-1.54$</td>
<td>$-4.89$</td>
</tr>
<tr>
<td>$</td>
<td>\Delta DCF</td>
<td>$ (%)</td>
<td>$10.26$</td>
<td>$-1.06$</td>
<td>$7.21$</td>
</tr>
</tbody>
</table>

Panel B: DCF valuation specifications

<table>
<thead>
<tr>
<th></th>
<th>Sell</th>
<th>Sell/Hold</th>
<th>Hold</th>
<th>Sell/Hold Buy</th>
<th>Sell/Hold Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ (years)</td>
<td>8.59</td>
<td>0.11</td>
<td>8.62</td>
<td>0.69</td>
<td>8.79</td>
</tr>
<tr>
<td>$g_2$ (%)</td>
<td>1.67</td>
<td>0.10</td>
<td>1.69</td>
<td>1.27</td>
<td>1.82</td>
</tr>
<tr>
<td>$TV/EV$ (%)</td>
<td>62.53</td>
<td>0.01</td>
<td>62.52</td>
<td>1.39</td>
<td>64.01</td>
</tr>
<tr>
<td>$h_T$</td>
<td>15.12</td>
<td>-0.74</td>
<td>13.61</td>
<td>$-2.27^{**}$</td>
<td>10.83</td>
</tr>
<tr>
<td>$\Delta TP$ (%)</td>
<td>2.82</td>
<td>0.78</td>
<td>4.09</td>
<td>4.14***</td>
<td>8.15</td>
</tr>
</tbody>
</table>

Panel C: DCF valuation performance

<table>
<thead>
<tr>
<th></th>
<th>Sell</th>
<th>Sell/Hold</th>
<th>Hold</th>
<th>Sell/Hold Buy</th>
<th>Sell/Hold Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NGET$</td>
<td>24</td>
<td>169</td>
<td>267</td>
<td>460</td>
<td></td>
</tr>
<tr>
<td>$NGET$ (%)</td>
<td>32.50</td>
<td>7.30***</td>
<td>72.22</td>
<td>$-4.39^{***}$</td>
<td>56.80</td>
</tr>
</tbody>
</table>

$TP - P_0$ is the difference between the target price ($TP$) of the equity report and the stock price ($P_0$) at date of report publishing. $\Delta DCF$ and $|\Delta DCF|$ are the difference between target prices published in the equity reports and target price using only the DCF method. $NGET$ is the number of target prices published in the equity reports that were reached by the stock market prices in the twelve months after the date of publication. Stock prices are from Datastream. The significance levels for the tests on the difference are based on $t$-statistics (average) and $Z$-tests of equal proportions for the $NGET$ sample proportions. Significance level at $^{***1\%}$, $^{**5\%}$ and $^{*10\%}$.
for the Sell-Buy pair is statistically different to zero). With increasing level of recommendation, the degree with which the equity report target price reflects that of the DCF method increases.

Panel B of Table 2 shows the average DCF specifications for class of recommendation. The average values of both the length of the explicit forecast period and the steady-state growth rate $g_2$ are almost constant with the variation in the level of the recommendation. The weight of the terminal value over the estimated equity value and the variable $\Delta TP\%$ both present higher values for the recommendation Buy, while the coefficient of reinvestment for the terminal year has a decreasing trend that varies according to the recommendation class. It therefore seems that the most positive valuations are associated with a greater difference in the convergence conditions between return and cost of capital.$^7$

Lastly, in Panel C of Table 2 the performance of the valuations is measured in terms of ability to forecast the stock price. The accuracy of a valuation is therefore measured as the ability to match the future stock price. In particular, when the target price is greater than the current trade price, the valuation is considered to be accurate if, on any day in the following 12 months from the date of publication of the report, the share price exceeds the target level. Similarly, for a target price below the current price, the valuation is considered accurate if the share price falls below the target price. To this extent, the dichotomous variable $GET$ verifies that the shares have effectively reached the target price during the 12-month time period.$^8$ The dummy variable $GET$ is therefore defined as equal to 1 if the actual trade price reaches the target price within 12 months of publication of the equity report, 0 otherwise.

The aim of the study is to verify the effective usefulness of the index $\Delta TP\%$ as a possible index of accuracy for DCF valuations. The efficacy of $\Delta TP\%$ as an accuracy index would be confirmed if it were negatively related to the probability that the target price is achieved by the stock price ($GET$). Should this be the case, a less plausible target price corresponds to a larger $\Delta TP\%$ value.$^9$ Table 2 reports the variable $NGET$ defined as the frequency of $GET$ (i.e., equity reports whose target prices are effectively reached by the share price). Most of the target prices are achieved within 12 months (58.9 per cent) and predictably, in particular for the class of reports recommending Hold (77.2 per cent), followed by Buy with 56.8 per cent and Sell with 32.5 per cent. Therefore, the empirical evidence shows how in reports with negative recommendations the target price were on average too conservative and achieved with a lower probability than in reports with more favorable opinions.

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The basic assumption of our approach is the pure-competition setting for the long-term steady state. Economic theory suggests indeed that competition will eliminate eventual abnormal returns in the long-run, so that the incremental return on capital in the continuing value period will equal the cost of capital. However, one could think that there are several markets that have elements of oligopoly leading to longer-term rents (i.e., \( \text{ROIC} > \text{WACC} \)). This applies equally to almost any business that sells something proprietary that is unlikely to be duplicated. Moreover, even within competitive equilibrium, accounting conservatism can lead to ROIC that is different from WACC in steady state (Penman, 2007). Analysts may indeed be overly conservative because of the uncertainty and size of the continuing value; hence, conservatism may overcompensate for the uncertainty.

These considerations do not invalidate our approach. Our contribute is to provide an accuracy index measuring the variance of the target prices due to variations in long-term assumptions that are not coherent with steady-state assumptions of pure competition. Long-term expectations on a particular firm may deviate toward a positive ROIC versus WACC spread only on the base of structural (long-term sustainable) competitive advantage. This means that the firm is expected to hold market (oligopoly) power for an indefinite period.\(^{10}\)

5. Empirical Validation: Regression Analyses

The effectiveness of the accuracy index \( \Delta TP\% \) was analyzed using a model aimed at verifying its capacity to identify those valuations performing better in terms of probability of reaching the target price. To do this, two types of regression analysis are used. Firstly, a Logit regression model was used that expresses the \( \text{GET} \) variable as a function of the variance index and other control variables. Control variables were chosen from those defined in Table 2 or in Appendix D as they are expected to play a role in the probability of reaching the target price. For instance, in the regression we include the variable \( |TP - P_0|/P_0 \) as it is expected to be negatively correlated to the \( \text{GET} \) probability. Appendix D lists firm-specific control variables such as firm size, trading levels, and price-momentum, and provides information on the calculation method and reference bibliography.

In addition to the Logit regression analysis, the typical techniques for analyzing censored data were used, particularly the Cox risk proportional regression model. Therefore, not only the probability of reaching...
the target price (GET variable) is taken into consideration as a dependent variable in the model, but also the time elapsing between the publication of the report and the (eventual) achievement of the target price (variable TIME_TO_GET).

5.1 Logit Regression

A logistic regression with GET as dependent variable was used in order to analyze the accuracy of DCF valuations, which tests the effectiveness of the variance index ΔTP% as a measure of valuation accuracy. The final model includes four firm-specific control variables: the size of the company (SIZE), the level of trading (TURN), momentum (PM), and analyst coverage (COVER). Other control variables are |ΔDCF|, that is, the absolute difference between target prices published in the equity reports and target price using the DCF method only, |TP − P₀|/P₀, that is the gap between target price and current price, and the dummy variable D_Hold, which has a value of one for the valuations with recommendation Hold.

The regression used is reported below, while the results are shown in Table 3.

\[
\text{logit}(\pi) = \alpha + \beta_1 \Delta TP + \beta_2 |TP − P_0|/P_0 + \beta_3 |\Delta DCF| + \beta_4 \text{SIZE} \\
+ \beta_5 \text{TURN} + \beta_6 \text{PM} + \beta_7 \text{COVER} + \beta_8 D_{\text{Hold}} + \beta_9 \text{NEG} + \varepsilon
\]

where \(\pi\) is the probability that GET has a value of 1. If the coefficient \(\beta_i\) is positive, an increase in the explicative variable \(i\) causes an increase in the probability that GET is equal to 1.

The analysis is conducted both on the whole sample and on the sub-sample of equity reports whose target price is greater than the current share price. The regression on the sub-sample focuses the analysis on the usefulness of the explicative variable ΔTP% in identifying overoptimistic valuations. The variable dummy NEG is included in the regression of the whole sample so as to differentiate other valuations that provide a lower target price than the current share price.

As expected, the variable ΔTP% is negatively related to the probability of achieving the target price (1 per cent significance level), with reference to both the entire sample and the sub-sample with target price greater than the current share price. This means that the probability of reaching the target price decreases with the increase in the variance between using the ideal long-term growth \(g^*_2\) (determined by the assumption
of asymptotic equivalence between cost and return on capital) and the real long-term growth used in the equity report.

Concerning control variables, the most explicative one is the difference between target and current price. The probability that the target price is reached obviously depends on how optimistic (or pessimistic) the valuation is and decreases in proportion to the size of the difference between target and current price (Asquith et al., 2005). The variables PM and TURN are significant and as expected TURN is negative and PM is positive. The pre-report turnover (TURN) is therefore negatively correlated to the probability of achievement of the target price. One possible explanation can be deduced from the Lee and Swaminathan (2000) hypothesis that the volume of trades is inversely correlated to future returns, since shares with low/high levels of trading are under/overvalued by investors. Furthermore, Jegadeesh et al. (2004) find that the most favorable valuations are expected for shares with a low turnover. On the other hand, the size of the firm (SIZE) and the analysts’ coverage (COVER) variables in this sample are not statistically significant. The variable D_Hold expresses the greater probability of reaching the target price for shares with a recommendation Hold (1 per cent significance level; also see the descriptive statistics in Table 2); while the variable NEG shows that target prices lower than current prices have a lower probability of being reached (1 per cent significance level). The variable

Table 3. Result of the Logit regressions on the probability of GET

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>Reports with TP &gt; P₀</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficients</td>
<td>z-Statistics</td>
</tr>
<tr>
<td>Constant</td>
<td>1.72</td>
<td>3.68***</td>
</tr>
<tr>
<td>ΔTP%</td>
<td>-2.40</td>
<td>-3.51***</td>
</tr>
<tr>
<td></td>
<td>TP – P₀</td>
<td>P₀</td>
</tr>
<tr>
<td></td>
<td>DCF</td>
<td>-3.05</td>
</tr>
<tr>
<td>SIZE</td>
<td>0.05</td>
<td>0.73</td>
</tr>
<tr>
<td>TURN</td>
<td>-0.09</td>
<td>-2.11**</td>
</tr>
<tr>
<td>PM</td>
<td>0.66</td>
<td>2.07**</td>
</tr>
<tr>
<td>COVER</td>
<td>0.06</td>
<td>0.61</td>
</tr>
<tr>
<td>D_Hold</td>
<td>0.797</td>
<td>4.03***</td>
</tr>
<tr>
<td>NEG</td>
<td>-2.09</td>
<td>-7.33***</td>
</tr>
<tr>
<td>No Obs.</td>
<td>772</td>
<td></td>
</tr>
<tr>
<td>Pseudo R²</td>
<td>11.96%</td>
<td></td>
</tr>
<tr>
<td>χ²</td>
<td>124.66***</td>
<td></td>
</tr>
</tbody>
</table>

See Table 2 and Appendix D for definition of the variables. D_Hold is a dummy variable to identify valuations with Hold recommendations. The variable dummy NEG was included in the regression of the whole sample in order to differentiate the valuations with a target price lower than the current stock price.
$|\Delta \text{DCF}|$ is negatively related to the probability of achieving the target price. This suggests that the growth in absolute terms of the difference between equity report target price and DCF price implies a lower probability of reaching the desired price.

### 5.2 Cox Regression

The Logit regression model estimates the accuracy of a valuation by considering whether or not the target price indicated in the equity report is achieved within 12 months. Consequently, a typically adopted period of 1 year from the report publication date is defined within which the market price forecasts in the valuation must be reached. However, this procedure does not consider time. A valuation whose target price is reached in the weeks immediately following the report publication cannot be distinguished from one in which the share price reaches the target price 12 months later. Nevertheless, since an optimal period for reaching the target price cannot be defined, it would also be useful to investigate this phenomenon from a temporal perspective. Therefore, not only the achievement of the target price, but also the time required to reach it is considered. Hence, a variable \((\text{TIME\_TO\_GET})\) was defined that measures the time elapsing between publication of the report and the (possible) achievement of the target price.

A duration model is used for the regression in which the dependent variable assumes two dimensions: time taken to achieve the target price \((\text{TIME\_TO\_GET})\) and the achievement dummy \((\text{GET})\). Duration models were originally used in epidemiological studies to study the effectiveness of medical treatments. Their use is also widespread in engineering, for example in industrial life testing studies, and also since the 1980s in economic studies such as on the probability of bankruptcy (Lane et al., 1986) and in employee turnover (Somers, 1996). This paper is the first study to adopt this methodology in financial accounting.\(^{11}\)

In the model presented in this paper, the event determining the transition of state is the share price achieving its target price. It follows that the “survival time” \(T\) is defined as the time required to achieve this, while its distribution function defines the so-called “failure function”, that is, the probability of reaching the target price by a certain day \(t\) after publication of the report: \(F(t) = \Pr(T < t)\), where \(t\) is the number of days since publication of the report and \(T\) the day the target price will possibly be reached. The corresponding density function is therefore \(f(t) = dF(t)/dt\), while the complementary function identifies the “survival function”: \(\prod\).
Lastly, the “hazard function” \( h(t) \) is defined as the probability that the achievement of the target price will occur during the period \( t + \Delta t \) (with \( \Delta t \) approaching to zero), subject to time \( t \) having already been reached: \( h(t) = F(t)/S(t) = f(t)/(1 - F(t)) \). These functions are used to estimate the impact of independent variables (such as the index \( \Delta TP\% \)) on the probability and on the time required for the transition of state, which in this study occur at the achievement of the target price. Figure 1 shows the failure function, with approximately half of the target prices being reached in the first 6 months following publication of the report, while a further 10 per cent approximately are achieved in the following 6 months. In order to graph the explanatory role of the index \( \Delta TP\% \), the hazard function is estimated by subdividing the sample by the value of \( \Delta TP\% \). The Failure Function for reports with a \( \Delta TP\% \) greater than 20 per cent lies below that estimated for the other sub-samples (Figure 2). This means that valuations with a higher value of \( \Delta TP\% \) have a lower and “lagged” probability of reaching the target price.

Figure 2 graphs the smoothed hazard function estimated using the Weighted Kernel density estimate. The graph suggests how the hazard rate grows during the first weeks after publication of the equity report and then gradually declines.
A Cox (1972) regression model is used to estimate the effects of the index \( \Delta TP\% \) and any other covariates on the hazard function. The same independent variables previously used in the logistical regression are also employed for the Cox model, with the addition of a dummy variable \( D_{Sell} \) that identifies the valuations with Sell recommendations. The estimated model is the following:

\[
h_i(t) = h_0(t)g_i(t) = h_0(t)\exp(X_i\beta)
\]

where \( h_0(t) \) is defined as baseline hazard function.

The results of the Cox model confirm the negative relationship between \( \Delta TP\% \) and the probability of \( GET \) (Table 4). In particular, the hazard ratio estimated for the whole sample is 0.25 per cent. This means that a valuation with \( \Delta TP\% = 1 \) has a 25 per cent probability of reaching the target price compared with an equity report with \( \Delta TP\% = 0 \). The effectiveness of the accuracy index is therefore confirmed by the duration model. Indeed, the results obtained using the Cox regression, as with those from the Logit, seem to strengthen the effective usefulness of the accuracy index \( \Delta TP\% \).

As far as the control variables are concerned, the one with the greater explicative power is again the difference between target price and current share price. The regression was also run on the sub-sample of equity reports with target prices higher than current prices. In this case, the variables \( NEG \) and \( D_{Sell} \) are eliminated as in excluding the valuations with target prices below the current price, these dummies lose their...

\[\text{Figure 2. Smoothed hazard function.}\]
explicative powers. The hazard ratio for the $\Delta TP\%$ of the sub-sample is similar to that found for the full sample (0.27 vs 0.25), while the Likelihood Ratio increases.

6. Conclusions

In the context of the recent international debate on the formal equivalence of valuation models and their empirical comparison, the present study proposes an accuracy index that verifies the coherence of the long-term assumptions implicit in DCF valuations. Relating the DCF method to the concept of RI, the steady-state assumptions at the base of long-run forecasts are highlighted. An ideal constant growth rate of cash flows is defined for which there are no opportunities for extra-profitability in the long-term. In these conditions, the rate of perpetual growth in the final stage of the DCF is equal to the product of cost of capital (in general, WACC) and the coefficient of reinvestment estimated for the final year of the period of implicit forecasts. Such an ideal growth rate reflects the asymptotic equivalence between profitability and cost of capital. Therefore, using this “ideal” growth rate leads to the determination of “ideal” target prices that respect the long-term steady-state assumptions.

This study compares the target prices published in equity reports to the “ideal” ones and defines a variance index between these two prices.

---

Table 4. Result of the Cox regressions on the probability of GET

<table>
<thead>
<tr>
<th></th>
<th>Coefficients</th>
<th>Hazard Ratios</th>
<th>z-Statistics</th>
<th>Coefficients</th>
<th>Hazard Ratios</th>
<th>z-Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta TP%$</td>
<td>-1.37</td>
<td>0.25</td>
<td>-3.58***</td>
<td>-1.31</td>
<td>0.27</td>
<td>-3.21***</td>
</tr>
<tr>
<td>$(TP - P_0)/P_0$</td>
<td>-3.21</td>
<td>0.04</td>
<td>-6.35***</td>
<td>-4.29</td>
<td>0.01</td>
<td>-7.41***</td>
</tr>
<tr>
<td>$</td>
<td>\Delta DCF</td>
<td>$</td>
<td>-1.38</td>
<td>0.25</td>
<td>-2.63***</td>
<td>-1.87</td>
</tr>
<tr>
<td>$SIZE$</td>
<td>0.02</td>
<td>1.03</td>
<td>0.54</td>
<td>0.04</td>
<td>1.04</td>
<td>0.76</td>
</tr>
<tr>
<td>$TURN$</td>
<td>-0.04</td>
<td>0.95</td>
<td>-1.67*</td>
<td>-0.06</td>
<td>0.94</td>
<td>-2.19**</td>
</tr>
<tr>
<td>$PM$</td>
<td>0.66</td>
<td>1.94</td>
<td>3.43***</td>
<td>0.85</td>
<td>2.34</td>
<td>4.14***</td>
</tr>
<tr>
<td>$COVER$</td>
<td>0.05</td>
<td>1.05</td>
<td>0.83</td>
<td>0.02</td>
<td>1.02</td>
<td>0.30</td>
</tr>
<tr>
<td>$D_{Hold}$</td>
<td>0.43</td>
<td>1.54</td>
<td>3.57***</td>
<td>0.38</td>
<td>3.05</td>
<td>3.57***</td>
</tr>
<tr>
<td>$D_{Sell}$</td>
<td>-0.95</td>
<td>0.39</td>
<td>-3.11***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$NEG$</td>
<td>-1.5</td>
<td>0.39</td>
<td>-7.51***</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No Obs. 772 650
No GET 460 405
$\chi^2$ 198.12*** 204.9***

See Table 2 and Appendix D for definition of the variables. $D_{Hold}$ and $D_{Sell}$ are dummy variables used to identify evaluations with Hold and Sell recommendations.
valuation that respects the steady-state conditions shows a null variance, while positive variances are associated with valuations in which the infinite-horizon stage of the forecasts is defined in imperfect steady-state conditions for the company.

The theoretical arguments are empirically validated with reference to a sample of 784 equity reports for European companies selected from the Investext database in the 3-year period 2003–2005. The accuracy of a valuation is measured in terms of probability of its target price being reached from the relative stock price. Using Logit and Cox regression methods, it is shown that the accuracy index proposed in the paper correlates significantly with performance of the valuation. In other words, equity reports that respect the long-term steady state are associated with a greater probability of reaching the target price during the 12 months succeeding the publication of the report. Therefore, the results suggest the accuracy index is valid as it identifies valuations with a greater probability of the actual trade price reaching the target price.

We expect this research to be interesting, both from an academic and from a practitioner standpoint. The current study is the first to investigate the importance of steady-state modeling in cash flow valuation models and to provide large-sample empirical evidence. Implementation guidance for cash flow valuation models are desirable for analysts, especially for the continuing value. To this extent, we provide a means of measurement that indicates how accurate DCF valuations are in relation to the long-term sustainability of the hypotheses implicit in the valuation. Part of the appeal of the study is its immediate applicability to valuation practice.

Notes

1. “The practical issue is what accounting—cash accounting in discounted cash flow models or accrual accounting in so-called RI models—best provides a base to which such a growth rate can be applied” (Penman, 2001, p. 685). “This challenge of finding appropriate continuing value expressions will remain a very hot topic for accounting based valuation researchers for the foreseeable future” (Richardson and Tinaikar, 2004, p. 246).

2. According to the assets-side version of the DCF model, cash flows to the firm (FCFF) are discounted at a rate that takes into account the remuneration required by all the categories of holders of the firm, usually estimated in terms of WACC.

3. A similar definition is proposed by Damodaran (2006). In Appendix C we report a “how-to-do-it” application with two analyst valuations for individual firms.

4. Reports on financial and insurance companies were excluded because the DCF method for valuing these types of company is not only rarely used, but also presents various peculiarities compared with those used for industrial companies. Cash flow forecasts should give details on the Net Operating Profit net of unlevered taxes and the component attributable to reinvestments. The sample of equity valuation is from Investext database, the world’s largest online database of equity reports and industry researches.
provides the full text of 2 million company, industry, and geographic research reports written by analysts at more than 600 leading investment banks, brokerage houses and consulting firms worldwide. Investext offers the complete text of each research report and abstracts for most reports; coverage is from July 1982 through 2008. We selected the publishing period 2003–2005 for which 153,204 reports are indexed at online page level in Investext. Most of these are actually industry or geographic reports, rather than firm valuations. Using software-assisted content analysis, which consists in extracting information from plain text, we find 2,320 equity reports that specify target price estimates and use the DCF model to value a European firm. However, far from all analyst reports contain enough details for our purposes. From this original sample, we essentially hand-collect analyst reports that contain enough forecasts to build a multi-year cash flows model. We acknowledge that it is probable that analysts who publish full financial forecasts are “fundamentals-oriented” analysts, and therefore more disciplined in their assessment of firms, target prices, etc., than the “average” analyst. Therefore, the magnitude of the target price variance in our research is likely even higher in the population. We thank the referee for comments on this strong-point in our research design and for inducing us to pay more attention to the description in the paper of the process used select the final sample of analysts’ reports.

5. The analyses presented in this paper were repeated by segmenting the sample by year of report publication. No significant year-specific effects were identified. Sometimes a three-stage DCF is used where the first period of explicit forecast is followed by a second phase where only the main economic-financial features are explicitly forecasted. This represents an attempt to overcome the trade-off between the need to extend the stage of explicit forecast long enough, and on the other hand, the need to limit the explicit forecast period to a reasonable length so that a reliable estimate is obtained. Three-stage DCF typically improve the identification of the steady state for the beginning of the last period of implicit forecasts.

6. The choice is justified by the fact that this structure is used in most reports in the sample and scales with five levels are only seen occasionally (14 cases). To harmonize the five level structure with the others, the two most negative recommendations were classified as Sell, the intermediate opinion as Hold, and the two most favourable as Buy.

7. ΔTP\% can also be higher because of the lower rate of investment h_T, which results in a reduced ideal g_2, quite a different figure from that calculated in the report. At the same time, a lower h_T for the higher ratings may reflect an increase in the Terminal Value thanks to the increase in perpetual cash flow relating to over-optimism concerning the value of the company.

8. To limit the effect of price volatility, the daily price was not used but the 30-day moving average instead. Therefore, a valuation is considered correct if the 30-day moving average of the company stock price reaches the target price. This technique is expected to improve the capacity to identify those valuations that “really” reach the target price from those that reach the target only occasionally. The results of the succeeding analyses throughout this paper refer to moving average post-publication stock prices. However, whether using raw stock prices or moving averages, there were very few differences in the GET variable.

9. In order to maintain interpretative coherence in the analyses, the variable ΔTP\% changed sign for observations in which the target price is less that the current share price. Moreover, through the effect of normal price volatility, valuations with a target price closer to the current share price have a greater probability of being accurate. Consequently, the effect of the level of optimism (or pessimism) of the valuation needs to be considered. To this extent, we define the variable |TP – P_0|/P_0 as the percentage difference in absolute value between target price and current price. Furthermore, the volatility of the share can influence the accuracy of the valuation in that more volatile...
shares might have a greater probability of reaching the target price. *Vice versa*, it is also possible that a greater level of volatility might be associated with a larger number of valuation nuisances, and consequently a lower rate of reaching the target price, as verified by Bradshaw and Brown (2005). In order to minimize this effect, the authors of this paper argue that the time series of stock prices needs to be smoothed by using a 30-day mobile average.

10. Our study provide large-sample empirical evidence on the importance of steady state modeling in DCF real—world valuations. We cannot therefore specify different cross sectional assumptions for the market power of individual firms. However, at an industry level, we note that higher average levels of \( \Delta TP\% \) are more common in heavily concentrated industries, such as utilities, resources, transports and pharmaceuticals. Further investigations in this direction must necessarily take into account the degree of oligopoly power at micro-industry level. It is indeed plausible to expect non-competitive mark-ups in more concentrated micro-industries (Herfindahl index). For instance, in the food-processing sector, a myriad of studies have found that some micro-industries (such as cocoa or sugar refining) are non-competitive (Lopez et al., 2002). Unfortunately, our subsample of 28 companies active in this sector does not provide empirical support.

11. For a detailed explanation of duration models see Kalbfleisch and Prentice (2002).

12. The Kaplan–Meier product-limit method is used to estimate the survival function, the Nelson–Aalen method for integrated hazard function, and the Weighted Kernel density for the smoothed hazard function.

13. Using a Cox model does not need a specific functional form for \( h_0(t) \) to be hypothesized. However, if models are used that need to hypothesize a specific functional form of the hazard function, similar results are found: the hazard ratio of the index \( \Delta TP\% \) estimated using an exponential model is 0.23, while using a Weibull model it is 0.24, and with a Gompertz it is 0.25.

References


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Appendix A: Proof of Equation (2)

Operating Asset Relation:

\[ \Delta IC_t = NOPAT_t - FCFF_t \]
\[ FCFF_t = NOPAT_t - \Delta IC_t \]

DCF model:

\[ EV_t = \sum_{i=1}^{\infty} \frac{FCFF_{t+i}}{(1 + WACC)^i} \]
\[ EV_t = \sum_{i=1}^{\infty} \frac{NOPAT_{t+i} - \Delta IC_{t+i}}{(1 + WACC)^i} \]
RI Model:

\[ RI_t = (ROIC_t - WACC) \cdot IC_{t-1} = NOPAT_t - WACC \cdot IC_{t-1} \]

\[ NOPAT_{t+i} = RI_{t+i} + WACC \cdot IC_{t+i-1} \]

RI and DCF model:

\[ EV_t = \sum_{i=1}^{\infty} \frac{NOPAT_{t+i} - \Delta IC_{t+i}}{(1 + WACC)^i} EV_t \]

\[ = \sum_{i=1}^{\infty} \frac{RI_{t+i} + WACC \cdot IC_{t+i-1} - \Delta IC_{t+i}}{(1 + WACC)^i} \]

\[ EV_t = \sum_{i=1}^{\infty} \frac{RI_{t+i} + WACC \cdot IC_{t+i-1} - IC_{t+i} + IC_{t+i-1}}{(1 + WACC)^i} \]

\[ EV_t = \sum_{i=1}^{\infty} \frac{IC_{t+i-1} \cdot (1 + WACC) - IC_{t+i} + RI_{t+i}}{(1 + WACC)^i} \]

\[ EV_t = \sum_{i=1}^{\infty} \frac{IC_{t+i-1} \cdot (1 + WACC)}{(1 + WACC)^i} - \sum_{i=1}^{\infty} \frac{IC_{t+i}}{(1 + WACC)^i} + \sum_{i=1}^{\infty} \frac{RI_{t+i}}{(1 + WACC)^i} \]

\[ EV_t = \left[ \sum_{i=1}^{\infty} \frac{IC_{t+i-1}}{(1 + WACC)^{i-1}} \right] - \sum_{i=1}^{\infty} \frac{IC_{t+i}}{(1 + WACC)^i} + \sum_{i=1}^{\infty} \frac{RI_{t+i}}{(1 + WACC)^i} \]

\[ EV_t = IC_t + \sum_{i=1}^{\infty} \frac{IC_{t+i}}{(1 + WACC)^i} - \sum_{i=1}^{\infty} \frac{IC_{t+i}}{(1 + WACC)^i} + \sum_{i=1}^{\infty} \frac{RI_{t+i}}{(1 + WACC)^i} \]

\[ EV_t = IC_t + \sum_{i=1}^{\infty} \frac{RI_{t+i}}{(1 + WACC)^i} \]
Appendix B: Proof of Equation (3)

Growth rate of NOPAT:

\[ g_{t}^{NOPAT} = \frac{NOPAT_t - NOPAT_{t-1}}{NOPAT_{t-1}} \]

\[ g_{t}^{NOPAT} = \frac{ROIC_t \cdot IC_{t-1} - ROIC_{t-1} \cdot IC_{t-2}}{NOPAT_{t-1}} \]

\[ g_{t}^{NOPAT} = \frac{ROIC_{t}^{old} \cdot IC_{t-2} + ROIC_{t}^{marg} \cdot \Delta IC_{t-1} - ROIC_{t-1} \cdot IC_{t-2}}{NOPAT_{t-1}} \]

\[ g_{t}^{NOPAT} = \frac{(ROIC_{t}^{old} - ROIC_{t-1}) \cdot IC_{t-2} + ROIC_{t}^{marg} \cdot \Delta IC_{t-1}}{NOPAT_{t-1}} + \frac{ROIC_{t}^{marg} \cdot \Delta IC_{t-1}}{NOPAT_{t-1}} \]

\[ g_{t}^{NOPAT} = \frac{ROIC_{t}^{old} - ROIC_{t-1}}{ROIC_{t-1}} + \frac{\Delta IC_{t-1}}{NOPAT_{t-1}} \cdot ROIC_{t}^{marg} \]

\[ g_{t}^{NOPAT} = g_{t}^{ROICold} + h_{t-1} ROIC_{t}^{marg} \]

Steady-state conditions (Koller et al., 2005):

(i) The incremental return on new invested capital is constant during the steady state:

\[ ROIC_{t+i}^{marg} = ROIC_{t}^{marg} = \text{cost} \]

(ii) The investment rate (defined as net investment over operating profits) is constant during the steady state and it is equal to the investment rate at the final year of explicit forecast:

\[ h_{T+i} = h_{T} = \text{cost} \]

(iii) The incremental return on new invested capital is constant, so the average return on invested capital varies in the second stage only as a consequence of new investments:

\[ g^{ROICold}_{T+i} = 0 \]

As a consequence:

\[ g_{t}^{NOPAT} = g_{t}^{ROICold} + h_{t-1} ROIC_{t}^{marg} \]

\[ g_{t}^{NOPAT} = h_{T} ROIC_{t}^{marg} = g_{2} \]
As the profitability of the past investments is constant:

$$\text{ROIC}_{t+i} = \frac{IC_{t+i-2} \text{ROIC}_{t+i-1} + \Delta IC_{t+i-1} \text{ROIC}^{\text{marg}}}{IC_{t+i-1}}$$

$$\text{ROIC}_{t+i} = \frac{IC_{t+i-2} \text{ROIC}_{t+i-1} + IC_{t+i-2} h_T \text{ROIC}_{t+i-1} \text{ROIC}^{\text{marg}}}{IC_{t+i-2}(1 + h_T \text{ROIC}_{t+i-1})}$$

$$\text{ROIC}_{t+i} = \frac{\text{ROIC}_{t+i-1}(1 + h_T \text{ROIC}^{\text{marg}})}{1 + h_T \text{ROIC}_{t+i-1}}$$

$$\text{ROIC}_{t+i} = \frac{\text{ROIC}_{t+i-2}(1 + h_T \text{ROIC}^{\text{marg}})^2}{1 + h_T \text{ROIC}_{t+i-2} + h_T \text{ROIC}_{t+i-2}(1 + h_T \text{ROIC}^{\text{marg}})}$$

For $t = T$:

$$\text{ROIC}_{T+i} = \frac{\text{ROIC}_T (1 + h_T \text{ROIC}^{\text{marg}})^i}{1 + \sum_{j=1}^{i} h_T \text{ROIC}_T (1 + h_T \text{ROIC}^{\text{marg}})^{j-1}}$$

$$\text{ROIC}_{T+i} = \text{ROIC}_T$$

$$\cdot \left[ \frac{1}{(1 + h_T \text{ROIC}^{\text{marg}})^i} + \frac{h_T \text{ROIC}_T}{1 + h_T \text{ROIC}^{\text{marg}}} \sum_{j=1}^{i} (1 + h_T \text{ROIC}^{\text{marg}})^{j-i} \right]^{-1}$$

$$\text{ROIC}_{T+i} = \text{ROIC}_T$$

$$\cdot \left[ \frac{1}{(1 + h_T \text{ROIC}^{\text{marg}})^i} + \frac{h_T \text{ROIC}_T}{1 + h_T \text{ROIC}^{\text{marg}}} \sum_{k=0}^{i-1} \frac{1}{(1 + h_T \text{ROIC}^{\text{marg}})^k} \right]^{-1}$$

$$\lim_{i \to \infty} \text{ROIC}_{T+i} = \lim_{i \to \infty} \left[ \frac{h_T}{1 + h_T \text{ROIC}^{\text{marg}}} \sum_{k=0}^{i-1} \frac{1}{(1 + h_T \text{ROIC}^{\text{marg}})^k} \right]^{-1}$$

$$\lim_{i \to \infty} \text{ROIC}_{T+i} = \text{ROIC}^{\text{marg}}$$
Assumption of long-term convergence between return and cost of capital:

\[ \text{ROI}^{\text{marg}} = \text{WACC} \]

\[ g_{t}^{\text{NOPAT}} = h_{T} \text{ROI}^{\text{marg}} = g_{2} \]

In ideal steady-state conditions:

\[ g_{2}^{\text{Ideal}} = \text{WACC} \cdot h_{T} \]

Appendix C: “How-To-Do-It” Application

The variance index compares how the target price derived from the DCF model using the long-term growth rate published in the analyst’s report \( (g_{2}) \) deviate from what the target price would have been using the “ideal” steady-state growth rate \( (g_{2}^{\text{Ideal}}) \). The Equity value of the firm \( (E) \) is equal to the difference between the Enterprise Value estimated using the DCF model \( (EV) \) and the net Debt \( (D) \); while the Target Price \( (TP) \) is the Equity value scaled by the number of shares outstanding.

\[ \Delta TP\% = \frac{TP - TP^{*}}{TP} = 1 - \frac{E^{*}}{E} = 1 - \frac{(EV - D)^{*}}{EV - D} \]  \( (4) \)

\[ \Delta TP\% = 1 - \frac{\sum_{i=1}^{T} \frac{FCFF_{t+i}}{(1 + WACC)^{i}} + \frac{FCFF_{T}}{(1 + WACC)^{T}} \cdot \frac{1 + g_{2}^{*}}{WACC - g_{2}^{2}} - D}{\sum_{i=1}^{T} \frac{FCFF_{t+i}}{(1 + WACC)^{i}} + \frac{FCFF_{T}}{(1 + WACC)^{T}} \cdot \frac{1 + g_{2}}{WACC - g_{2}} - D} \]

\[ = 1 - \frac{\sum_{i=1}^{T} \frac{FCFF_{t+i}}{(1 + WACC)^{i}} + TV^{*} - D}{\sum_{i=1}^{T} \frac{FCFF_{t+i}}{(1 + WACC)^{i}} + TV - D} \]

\[ \Delta TP\% = \frac{\frac{FCFF_{T}}{(1 + WACC)^{T}} \cdot \left[ \frac{1 + g_{2}}{WACC - g_{2}} - \frac{1 + g_{2}^{2}}{WACC - g_{2}^{2}} \right]}{\sum_{i=1}^{T} \frac{FCFF_{t+i}}{(1 + WACC)^{i}} + \frac{FCFF_{T}}{(1 + WACC)^{T}} \cdot \frac{1 + g_{2}}{WACC - g_{2}} - D} \]
The “ideal” steady state is defined as the state when a firm earns (in accounting terms) exactly its cost of capital, i.e., what we would expect in pure-competition settings. From this definition (essentially setting ROIC equal to WACC), we derive that the “ideal” perpetual growth rate \( g^*_2 \) to use in steady state (i.e., in terminal values) is equal to the reinvestment rate \( h_T \) times WACC.

\[
g^*_2 = WACC \cdot h_T = WACC \cdot \frac{\Delta IC_t}{NOPAT_t}
\]  

\[\Delta IC_t = NOPAT_t - FCFF_t \quad \text{[Operating Asset Relation]}\]

\[
g^*_2 = WACC \cdot \frac{NOPAT_t - FCFF_t}{NOPAT_t}
\]

The sample under investigation is made of different text-based analyst reports with slightly different modeling techniques. Nevertheless, we believe that a couple of applications of our approach to “real” analyst valuations could be helpful. We describe our approach with reference to two equity reports, an “average” case with a variance index close to the sample average (6.5 per cent), and an “extreme” case with a variance index equal to 53 per cent. The target price set in the first case was reached in the week after publication, while the stock price of the “extreme” case has never reached the target.

Analysts report 1: “average” case for which variance index is +6.5 per cent.

This equity report valuing a French food-product company was published on January 24, 2005. The DCF target price was €76.55 (€70 published as report’s target price) compared with €68 stock price at time. The Enterprise Value (€25,358M) was estimated with a two-stage DCF
model in which the 11-year long first stage accounted for 41 per cent of Enterprise Value (TV/EV equal to 59 per cent).

Accounting data at the end of the first stage \((T = 11)\) are as follows:

\[
\begin{align*}
\text{NOPAT}_{T=11} & = \€2,152M \\
\text{Depreciation} & = \€832M \\
\text{Capital Expenditures} & = \€1,108M \\
\text{Working Capital (change)} & = \€122M \\
\text{FCFF}_{T=11} & = \€1,755M \\
(NOPAT - FCFF)_{T=11} & = \€397M \\
h_t \left[\frac{(NOPAT - FCFF)}{NOPAT}\right]_{T=11} & = 18.45\%
\end{align*}
\]

Analysts’ assumptions on the long-term are as follows:

\[
\begin{align*}
\text{WACC} & = 7.42\% \\
g_2 & = 2\% \\
\text{TV} \text{ (Terminal Value, report)} & = \€15,029M \\
\text{EV} \text{ (Enterprise Value, report)} & = \€25,358M \\
\text{DCF TP} \text{ (Target Price, } DCF) & = \€76.55
\end{align*}
\]

Other variables (although not necessary to verify our approach):

\[
\begin{align*}
\text{IC}_{t=10} & = 13,712 \\
\text{IC} & = \text{operating working capital} + \text{net property, plant, and equipment} + \text{ other assets} \\
\text{ROIC}_{11} & = \frac{\text{NOPAT}_{11}}{\text{IC}_{10}} \quad 15.7\% \\
\text{RI}_{11} & = \left(\frac{\text{ROIC}_{11}}{\text{WACC}}\right) \frac{\text{IC}_{10}}{1,135}
\end{align*}
\]

\[
\lim_{i \to \infty} \text{ROIC}_{11+i} = \frac{\text{NOPAT}_{11}(1 + g_2)^i}{\text{IC}_{10} + \sum_{n=1}^{i} \text{NOPAT}_{11} h_T (1 + g_2)^{i-1}} = \frac{(1 + g_2)^i}{h_T \sum_{n=1}^{i} (1 + g_2)^{i-1}} = \frac{g_2}{h_T} = 10.84\% > \text{WACC}
\]

Our approach is based on analysts’ assumptions, but imposing an “ideal” growth rate \(g_2^*\), defined as the state when a firm earns exactly its cost of capital \((\text{ROIC}_{t=\infty} = \text{WACC})\).

\[
\begin{align*}
g_2^* & = \text{WACC} \cdot h_T \quad 1.37\% \\
\text{TV}^* \text{ (Terminal Value, ideal)} & = \€13,381M \\
\text{EV}^* \text{ (Enterprise Value, ideal)} & = \€23,710M \\
\text{TP}^* \text{ (Target Price, “ideal” } DCF) & = \€71.57 \\
\Delta \text{TP}\% & = +6.5\%
\end{align*}
\]
\[
\lim_{i \to \infty} ROIC_{11+i} = \frac{\text{NOPAT}_{11}(1 + g_2^*)^i}{\text{IC}_{10} + \sum_{n=1}^{i} \text{NOPAT}_{11} h_T (1 + g_2^*)^{i-1}}
\]
\[
= \frac{(1 + g_2^*)^i}{h_T \sum_{n=1}^{i} (1 + g_2^*)^{i-1}} = \frac{g_2^i}{h_T} = 7.42\% \equiv \text{WACC}
\]

Analysts report 2: “extreme” case for which the variance index is +53 per cent.

This equity report valuing a Dutch telecom company was published on November 10, 2003. The DCF target price was €2.30 (€2.25 published as report’s target price) compared with €1.70 stock price at time. The Enterprise Value (€1,153M) was estimated with a two-stage DCF model in which the 5-year long first stage accounted for 3.7 per cent of Enterprise Value (TV/EV equal to 96.3 per cent).

Accounting data at the end of the first stage \((T = 5)\) are as follows:

\[
\begin{align*}
\text{NOPAT}_{T = 5} & = 87.6M \\
\text{Depreciation} & = 158.4M \\
\text{Capital Expenditures} & = 192.7M \\
\text{Working Capital (change)} & = 16.4M \\
\text{FCFF}_{T = 5} & = 36.9M \\
(NOPAT - FCFF)_{T = 5} & = 50.7M \\
h_t [(NOPAT - FCFF)/NOPAT]_{T = 5} & = 58\%
\end{align*}
\]

Analysts’ assumptions on the long-term are as follows:

\[
WACC = 10.50\%
\]

The long-term growth rate \(g_2\) is not explicitly expressed in the report, as the terminal value is estimated as a multiple of operating margins in the terminal year. However, it can be easily back out from the Terminal Value estimated using multiples, \(WACC\) and \(FCFF_T\).

\[
TV = \frac{FCFF_T}{(1 + WACC)^T} \frac{1 + g_2}{WACC - g_2} \Rightarrow g_2
\]
\[
= \frac{WACC \cdot TV(1 + WACC)^T - FCFF_T}{TV(1 + WACC)^T + FCFF_T}
\]

Long-term \(EV/EBITDA\) 12.67%

\(g_2\) 8.3%

\(TV\) (Terminal Value, report) €1,110M

\(EV\) (Enterprise Value, report) €1,153M

\(DCF\ TP\) (Target Price, \(DCF\)) €2.30
\[
\lim_{i \to \infty} ROIC_{11+i} = \frac{NOPAT_{11}(1 + g^*_2)^i}{IC_0 + \sum_{n=1}^i NOPAT_{11}h_T(1 + g^*_2)^{i-1}}
\]

\[
= \frac{(1 + g^*_2)^i}{h_T \sum_{n=1}^i (1 + g^*_2)^{i-1}} = \frac{g^*_2}{h_T} = 14.34\% > WACC
\]

Our approach is based on analysts’ assumptions, but imposing an “ideal” growth rate \(g^*_2\):

\[
g^*_2 = WACC \cdot h_T
\]

\[TV^* (\text{Terminal Value, ideal}) \quad \text{€497M}\]

\[EV^* (\text{Enterprise Value, ideal}) \quad \text{€540M}\]

\[TP^* (\text{Target Price, “ideal” DCF}) \quad \text{€1.09}\]

\[\Delta TP \% \quad +53\%\]

\[
\lim_{i \to \infty} ROIC_{11+i} = \frac{NOPAT_{11}(1 + g^*_2)^i}{IC_0 + \sum_{n=1}^i NOPAT_{11}h_T(1 + g^*_2)^{i-1}}
\]

\[
= \frac{(1 + g^*_2)^i}{h_T \sum_{n=1}^i (1 + g^*_2)^{i-1}} = \frac{g^*_2}{h_T} = 10.50\% \equiv WACC
\]

### Appendix D: Description of Firm-Specific Control Variables

<table>
<thead>
<tr>
<th>Description of the variable</th>
<th>Prediction and reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(SIZE): Natural logarithm of market capitalization: (\ln(P \cdot \text{number of shares}))</td>
<td>It is a control variable for company size. Bradshaw and Brown (2005) find that the target price for large companies has a lower probability of being reached.</td>
</tr>
<tr>
<td>(TURN): Natural logarithm of daily turnover, calculated for the year preceding publication of the report.</td>
<td>Lee and Swaminathan (2000) show how the volume of trades is inversely related to the future performance; they argue that associated with high/low levels of trading are over/undervalued by investors. Thus, more favorable valuations are expected for low turnover shares (Jegadeesh et al., 2004).</td>
</tr>
</tbody>
</table>

\[
\ln \left( \frac{\sum_{i=1}^n P_i \cdot \text{number of traded shares}_i}{n} \right)
\]

\(n\): number of days of trades in the twelve months preceding the report publication date.
Appendix D. (Continued.)

<table>
<thead>
<tr>
<th>Description of the variable</th>
<th>Prediction and reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PM</strong>: Price Momentum, defined as cumulative share price performance relative to the market index. The following indices were used for the different markets: the FTSE 100 for the British market, CAC 40 for the French market, the DAX 30 for the German market, the MIB 30 for the Italian market, the AEX for the Dutch market, and the BEL 20 for the Belgian market.</td>
<td>This variable incorporates the price trend in relation to the general trend in the market. Jegadeesh and Titman (1993) document a positive correlation between Price Momentum of a stock and its performance in the succeeding twelve months; while Jegadeesh et al. (2004) show that the stocks with a positive momentum are those that receive the most favorable analysts’ recommendations.</td>
</tr>
<tr>
<td><strong>COVER</strong>: Number of reports on the company published in the Investext database in the year the report was published. It does not consider the number of investment banks that analyze the company, but only the total number of reports published.</td>
<td>This variable expresses the intensity with which analysts have studied the company (analysts’ coverage). Greater valuation accuracy is expected for the most analyzed companies due to the effect of a learning curve.</td>
</tr>
</tbody>
</table>

Appendix E: Notation

<p>| <strong>EV</strong> | Enterprise Value: asset side DCF models value the equity of a company as the value of a company’s operations (the enterprise value that is available to all investors) less the value of debt and other investor claims that are superior to common equity (such as preferred stock) |
| <strong>FCFF_t</strong> | Free Cash Flow to the Firm (at year t), equal to the after-tax operating earnings of the company, plus non-cash charges, less investments in operating working capital, property, plant and equipment, and other assets (it does not incorporate any financing-related cash flows such as interest expense or dividends) |</p>
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_t$</td>
<td>growth rate (at year $t$)</td>
</tr>
<tr>
<td>$g_2$</td>
<td>growth rate for the implicit forecast period (i.e., long-term growth rate used to estimate the Terminal Value)</td>
</tr>
<tr>
<td>$g_2^*$</td>
<td>“ideal” long-term growth rate calculated respecting the long-term steady-state assumptions: $WACC \cdot h_T$</td>
</tr>
<tr>
<td>$h_t$</td>
<td>investment rate, defined as net investment over operating profits (Koller et al., 2005: “this measure tells you whether the company is consuming more funds than it is generating (investment rate greater than one) or generating extra cash flow that can be paid to investors as interest expense, dividends, debt reductions, share repurchases, and so on”)</td>
</tr>
<tr>
<td>$IC_t$</td>
<td>Invested Capital (at year $t$), defined as operating working capital + net property, plant, and equipment + other assets</td>
</tr>
<tr>
<td>$NO-PAT_t$</td>
<td>Net Operating Profit less Adjusted Taxes (at year $t$), it represents the after-tax operating profits of the company after adjusting the taxes to a cash basis</td>
</tr>
<tr>
<td>$RI$</td>
<td>Residual Income, the spread between the return on capital and its opportunity cost times the quantity of invested capital: $(ROIC_t - WACC) \cdot IC_{t-1}$</td>
</tr>
<tr>
<td>$ROIC_t$</td>
<td>Rate of Return on Invested Capital (at year $t$), given from the ratio between the net operating profit after taxes and the invested capital: $ROIC_t = \frac{NOPAT_t}{IC_{t-1}}$</td>
</tr>
<tr>
<td>$WACC$</td>
<td>Weighted Average Cost of Capital, in which each category of capital is proportionately weighted (all capital sources—common stock, preferred stock, bonds and any other long-term debt—are included)</td>
</tr>
</tbody>
</table>