Equilibrium Distances of a Collinear Planetary System

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Abstract

Starting from Talbott, several mythologists have argued that during the initial phase of human history the solar system was radically different than now, the planets being aligned with the Sun during their revolution, with Saturn in a dominating position (the polar model). In this paper the planetary alignment equations introduced by Grubaugh and studied in a previous paper by Spedicato and Huang are modified by taking into account the baricenter of the system. The equations constitute a nonlinear underdetermined system of $m-1$ equations and m variables (the distances of the planets from the baricenter). The system is solved using Newton method as implemented in the MAPLE (release 2) package, after setting one of the variables (namely the distance of the Earth from the Sun) to a given value to obtain a determined system. A same solution is obtained to a very high residual accuracy in few iterations from several starting points, the difference with respect to the solution computed without taking into account the baricenter being almost negligeable. Additional parameters of interest are computed and discussed in their relation with the polar model. Overall the numerical results are in agreement with the polar model.

1. Introduction

Analysis of the invariant elements in essentially all religions and in the oldest mythologies worldwide has led several mythologists, notably Talbott (see *The Saturn Myth*, 1980), Cardona (see for instance *Let There Be Light*, Kronos III, 3, 1978) and Cochrane (see for instance *The Birth of Athena*, Aeon II, 3, 1990), to introduce the idea that during a remote time in the human experience our solar system was radically different than now. Such ancient configuration, called *the polar model*, some of whose features are recalled below, evolved into the present state after probably violent and dramatic events while Homo Sapiens was already living on Earth. This approach can be seen as a development of ideas already expressed even if not in a systematic way by Immanuel Velikovsky in his seminal monograph *Worlds in Collision* (Mc Millan, 1951). From an analysis of mainly biblical sources Patten (see *Catastrophism and the Old Testament*, 1986) has also introduced a different, albeit not so radically different, planetary scenario than now for the period from about 9500 BC to the year 701 BC. According to Patten during such a period Mars moved in an elliptical orbit and came very close to the Earth every 54 years with catastrophical effects (megacatastrophes, including the Flood, happened at longer intervals depending on the relative positions of the largest planets). Leaving to a future paper an attempt to conciliate a modified polar model with the Patten scenario, here we concentrate on the mathematical study of a feature of the polar model, namely the alignment of the planets with the Sun during their solar revolution.
The following are basic elements of the polar model:

- The Sun was not visible (this should be understood as from the part of the Earth where man was living).
- Saturn was the dominating object in the sky. It loomed large and his position in the sky appeared fixed.
- Venus appeared centered in the middle of Saturn and very bright. It was often referred to as the eye of Saturn.
- Mars appeared centered in the middle of Venus, but with variable angular size. It was of dark reddish colour and was often referred to as the pupil of the eye.
- No other planets, in particular Jupiter, were in the record.

The above features suggest the following physical state of the planetary system:

- A synchronous revolution of the visible planets, including the Earth, possibly along circular orbits, with the exception of Mars. Notice that synchronous circular orbits are a feature of gravitational systems with strong tidal coupling.

- The fixed position of Saturn is best explained if it is assumed that the Earth revolved around the Sun keeping the same face towards the Sun (as the Moon does with respect to the Earth and as is expected in gravitational systems with strong tidal coupling). The Earth's axes of revolution and of rotation would coincide and would be orthogonal to the ecliptic plane. The "day" of the Earth would be equal to the year.

- The nonvisibility of the Sun can be explained assuming that living conditions on the terrestrial hemisphere facing the Sun were impossible for man, for instance if such a hemisphere was completely covered by ocean. Hence man lived on the less hot hemisphere facing the external planets. One can conjecture that a large continent, most probably the classical Pangea whose breaking up originated the present continents, existed on this hemisphere.

- Nonvisibility of Mercury is explained as in the case of the Sun.

- Nonvisibility of the planets Jupiter, Uranus, Neptune can be explained either by their irrelevance, being much farther away, or by their location directly behind Saturn, if the planetary alignment applied also to them, or finally by their absence. It is tempting to hypothesize here that Jupiter has been recently captured and that this event led to the collapse of the polar configuration. Modelling and numerically studying the capture of Jupiter from the molecular cloud in the Orion region that was crossed by the solar system a few millions years ago is a work presently under way.

In this paper we only consider the equilibrium equations governing the planetary alignment, under the following assumptions:

- the planets are aligned

- the orbits are circular

- the only relevant forces are the gravitational forces and the masses are considered concentrated in their baricenter (therefore disregarding the important tidal forces)

- the planets are spheres with the same mass and size as now (therefore neglecting the likely loss of atmosphere during the assumed catastrophic capture of Jupiter, with probable reduction of the original planet diameter, and tidal effects probably resulting in non spherical shape of the planets)
- their order in the alignment is given
- the presence of planet satellites is ignored.

The equilibrium equations, essentially those considered by Grubaugh (Aeon, III, 3, 1993), but with the addition of the baricenter equation, are solved numerically using the Newton method. It appears that at least in a sizable region of the variable space they have a unique solution (a fact that is known theoretically to be true for the 3 body problem). We also compute related quantities of interest in the polar model, as angles of visibility and ratios of gravitational tidal forces, giving some discussion of their significance.

2. Equilibrium equations for a planetary alignment

Suppose that we have a system of m planets with masses $M_1, \ldots, M_m$ and a star of mass M_0, all revolving around their baricenter on circular orbits, with the same period and lying on a same line. Such a fixed geometry configuration is known to be feasible for the m-body pure gravitational problem and to be stable under certain conditions on the masses of the bodies and orbital parameters, see for instance Milani and Nobili (Celestial Mechanics 31, 213-240, 1983), even disregarding tidal effects, that generally contribute to the stability of the system (as is the case presently with the system Sun-Earth-Moon). Let the modulus of the distances of the considered planetary masses from the baricenter be $x_1, \ldots, x_m$. Let V_i be the modulus of the velocity of the ith planet. Then syncronicity and circularity of the orbits imply for every $i,\ j$, $(i,\ j = 1, \ldots, m)$:

$$V_i/x_i = V_j/x_j. \tag{1}$$

Under the given assumptions the gravitational force acting on the ith planet must be equal to the centripetal force. Hence we obtain, with x_0 the distance of the Sun from the baricenter and G the gravitational constant, for $i = 1, \ldots, m$

$$GM_i \left[\sum_{j=0}^{i-1} M_j/(x_i - x_j)^2 - \sum_{j=i+1}^{m} M_j/(x_i - x_j)^2 \right] = M_i V_i^2/x_i. \tag{2}$$

Cancelling M_i in (2) and using (1) to express V_i^2 in terms of, say, the Earth velocity V_E and distance x_E we obtain, for $i = 1, \ldots, m$

$$G\left[\sum_{j=0}^{i-1} M_j/(x_i - x_j)^2 - \sum_{j=i+1}^{m} M_j/(x_i - x_j)^2\right] = x_i V_E^2/x_E^2. \tag{3}$$

Equations (3) are a system of m equations in the $m+2$ variables x_i and V_E. We can get V_E from one of the equations (3), for instance from the Earth's equation, obtaining a system of $m-1$ equations for the $m+1$ variables x_i. Such equations have the following form, with E the index of the Earth in the given planetary sequence

$$\begin{aligned}&\left[\sum_{j=0}^{i-1} M_j/(x_i - x_j)^2 - \sum_{j=i+1}^{m} M_j/(x_i - x_j)^2\right] \\ &= \quad x_i/x_E\left[\sum_{j=0}^{E-1} M_J/(x_E - x_j)^2 - \sum_{j=E+1}^{m} M_j/(x_E - x_j)^2\right]. \end{aligned}\tag{4}$$

Notice that G has disappeared in (4), a welcome fact since the numerical value of G is still known only with about three digits, and that if we write the above system as $F(x) = 0$, then $F(\alpha x) = F(x)/\alpha^2$, hence the system is a homogeneous system of degree -2. Therefore if x is a solution, so is αx for any nonzero α. Finally a last equation is given, defining the baricenter

$$\sum_{j=0}^{m} x_j M_j = 0 \tag{5}$$

We are thus left with m equations in $m+1$ variables, defining an underdetermined nonlinear algebraic system with one degree of freedom. By fixing the value of one of the variables, namely the distance Earth to baricenter, the system will be reduced to a determined system. Notice that the addition of equation (5) invalidates the homogeneity of the whole system, since equation (5) is homogeneous of degree $+1$, not of degree -2. The system can be defined to be in a block homogeneous form and it is immediately seen that if x is a solution, the same is again true for αx for any nonzero α.

2. Solving the equations numerically

The given equations have been solved numerically for three planetary configurations. Configuration A consists of the planets Earth, Mars, Venus and Saturn in this order; configuration B has the Moon between Earth and Sun; configuration C has the Moon between Mars and Earth. Configuration D is the same as A, but the baricenter equation (5) is omitted. Configuration (E) has Jupiter beyond Saturn. In order to obtain a solution respecting the given configuration we use a change of variables. For configuration A let x_E, x_{MS}, x_V and x_S be the distances of Earth, Mars, Venus and Saturn from the baricenter. Then we define the variables $y_1, \ldots, y_4$ by relations

$$
\begin{aligned}
\mathrm{x}_E &= \mathrm{y}_1^2, \\
\mathrm{x}_{MS} &= \mathrm{x}_E + y_2^2 = y_1^2 + y_2^2, \\
\mathrm{x}_V &= \mathrm{x}_{MS} + y_3^2 = y_1^2 + y_2^2 + y_3^2, \\
\mathrm{x}_S &= \mathrm{x}_V + y_4^2 = y_1^2 + y_2^2 + y_3^2 + y_4^2.
\end{aligned}
$$

In a similar way for configurations B and C the position of the Moon is forced to lie respectively between Earth and Sun or between Mars and Earth and for configuration E Jupiter is forced to lie beyond Saturn.

By fixing a value of one of the variables, say x_E, equal to unity, the system becomes determined and can be solved numerically. Here the distance unit is assumed to be the astronomical unity (A.U.), corresponding to about 149.600.000 km., i.e. to the average distance of the Earth from the Sun. It can indeed be easily conjectured that the distance Earth to the Sun during the polar configuration could not be much different as now due to the presence of life on the Earth. Since life is dependent on the presence of liquid water, the distance could be only about at most 15% greater or smaller than now, since otherwise water would either completely freeze or vaporize (at least disregarding effects due to a possible different mass or atmospheric composition).

In the quoted paper by Spedicato and Huang a FORTRAN implementation of the Newton method was used, with the Jacobian matrix approximated by finite differences. The method converges on all problems from different starting points to an approximate solution having a Euclidean residual norm of about 10^{-10}, double precision having been used on a compatible 386 PC. In the present paper we use the version of the Newton method available in the package MAPLE V rel. 2. Here also double precision is used but the derivatives are computed analytically by simbolic differentiation. Initial values must be given to the variables to start Newton method. We used five different starting points. For configuration A we took $x_S = x_E + k/5x_E$, $k = 1, \ldots, 5$. We took $x_V = x_E + (k/5x_E)/3$, $x_{MS} = x_E + (k/5x_E)/4$. The computations were done in double precision on a compatible Pentium 133MH with zero machine about 10^{-20}. From all starting points Newton method converged in a few iterations to the same solution (close to the one obtained in the previous approach, where the baricenter equation was missing). While this is not a proof that the system has a unique solution (there are actually multiple solutions in the transformed space), this result can be taken as an indication that the solution may be unique in a rather large region of the variables space. Similarly defined starting points were used for the other configurations. The MAPLE code was run for 100 iterations and, as expected since analytical derivatives were used, gave a solution more accurate than that obtained by the previous finite difference implementation, the Euclidean norm of the final residual being of the order 10^{-13}.

3. The computed results

Table 1 gives the numerical results for all configurations, relating to the distances of the planet centers from the baricenter, having assumed that the distance Earth to the baricenter is equal to one (for Earth distances equal to α all other planetary distances follow approximately just by multiplying for α, from the homogeneity of equations (4)).

Table 1

Body	A	B	C	D	E
Sun	$-3.0035E-4$	$-3.0030E-4$	$-1.355E-5$	0	0
Earth	1	1	1	1	1
Moon		0.99328	1.003925		1.003365
Mars	1.00519	1.00517	1.005444	1.0052	1.004665
Venus	1.00988	1.00984	1.010039	1.0100	1.008593
Saturn	1.0522	1.015211	1.05228	1.0526	1.043757
Jupiter					1.124249

Remark 1 For case C the total tidal forces of the external bodies (Moon, Mars, Venus and Saturn) over the Earth are computed to be 7.97 times greater than the present tidal forces due to the Sun. For case B the total tidal forces are 7.73 times greater than the present tidal force due to the Sun. Hence the tidal forces in the considerd configuration are mainly given not by the Moon, as is the case now, but by the other planets, and are about 4 times greater than the present tidal forces due to the Moon.

Table 2 gives, under the headings α-Mars, α-Venus, α-Saturn, α-Moon, $\alpha - Jupiter$ the angles in degrees under which Mars, Venus, Saturn, Moon and Jupiter would be viewed from the Earth, assumed to be at one A.U. from the baricenter, for configuration E.

Table 2

α-Mars	α-Venus	α-Saturn	α-Moon	α-Jupiter
0.589	0.570	1.089	0.419	0.438

Remark 2 The above results do not change in their ratio if the distance Earth to baricenter is changed, in view of the homogeneity of the equations. It is observed that the visibility angle of Mars is greater than that of Venus, hence Venus would not be visible, falsifying a fundamental point of the *polar model*. However there is an escape from this fact, related to the fact that we have assumed as diameter of Venus the present diameter. If the collapse of the polar configuration went through a catastrophical phase involving loss of atmosphere of Venus, then we may argue that the original diameter of Venus was greater than now.

Remark 3 From the equilibrium equations one can compute also the modulus of the velocity of the considered bodies in their circular orbits. For the Earth, assumed at one A.U. from the baricenter, the speed is about 5.4 A.U., hence about 20% less than now (being presently 2π), or, in km/sec, about 24 km/sec. The speed is however very different than now for the other bodies. The fact that the computed speed of the Earth is not much different than now is quite remarkable, indicating that the collapsed system ended up in a new configuration where the important parameters relating to the Earth did not change much. This fact is certainly of crucial significance for the life on the Earth to have survived the hypothesized collapse.

4. Further analysis of the numerical results

The following observations also follow from an inspection of the Tables:

- in agreement with the Talbott *et al.* interpretation of the mythological record planet Saturn revolves quite close to the Earth. Its distance from the Sun is only about 5.3% greater than the Earth distance, if Jupiter is not considered (i.e.

about 8 million km), about 4.4% if Jupiter is considered. Jupiter would be only about 12% more distant than Earth (i.e. about 18 million km) and completely invisible, its visibility angle being smaller than the angles of Mars, Venus and Saturn

- Venus visibility angle is about the half of the Saturn angle, this value being essentially unaffected by the presence of Jupiter
- Mars visibility angle is about 7% greater than the Venus angle
- Saturn visibility angle would be greater than the present Moon's angle for x_E less than about 1.5 astronomical units
- gravitational tidal force of the external planets would be larger than the Sun's tidal force, the planets closest to the Earth producing the greatest force. Notice that the ratio of these forces is essentially independent of the actual Earth distance from Sun, due to the homogeneity property of equation (4). Notice that not only the ratio of the planetary tidal forces over the Sun's tidal force would be greater than the present ratio of Moon's over Sun's tidal force (about 2.2), but the resulting *action* would be greater if the Earth kept the same face towards the planets. This fact may have had remarkable influence on the geological structure of the Earth. It is tempting to associate the formation of a unique continental mass, the Pangea, with such a tidal action; it is moreover to be expected that the continuous action of the tidal force resulted in a substantial deformation of the Earth shape, towards probably an ovoid-type shape.

5. Conclusions

The polar model has been partially investigated under some assumptions. The numerical results do actually provide a quite satisfactory validation. Further work is necessary along the following lines:

- analysis of the dynamical stability of the circular syncronous orbits (i.e. time integration of the motion equations to verify if the alignment is stable; this may be true only for a range of values of x_E).
- analysis of the orbits in presence of a strong perturbation. It is of particular interest to investigate if the arrival of Jupiter as a body coming from outside the

previous solar system and passing presumibly close to Saturn and Venus could result in its capture, with the removal of Saturn to a farther away orbit, the passage of Venus into the present orbital region, after a likely grazing impact with Jupiter, and a perturbation of Mars' orbit into an elliptical one with strong eccentricity (this would result in the close and catastrophical interactions of Mars with Earth that have been considered by Patten).

- determination of the distance x_E of the Earth to the Sun at the time of the polar model configuration. This could be estimated, in addition to the stability argument given before, if mythological information could be obtained on the relative size of the visibility angle of the Moon versus the planetary angles, under the assumption that the Moon-Earth distance has not changed.

Of course the above problems are of considerable mathematical difficulty, both in the modelling and in the numerical solving, but should be within reach of computation of present workstations.

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