

# The relationship between industry localization and shift-share analysis

*La relazione tra struttura di localizzazione industriale e analisi shift-share*

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**Riassunto:** L'analisi shift-share viene comunemente utilizzata per lo studio delle dinamiche settoriali e territoriali. Tuttavia, i tassi di crescita specifici utilizzati per definire le componenti della scomposizione possono risentire della dimensione iniziale dell'aggregato, rendendo problematiche le analisi basate sul loro confronto (Biffignandi 2005). Per fronteggiare il problema, in questo lavoro viene proposta una modifica alla versione di Arcelus dell'analisi shift-share, in cui la componente di mix regionale viene scomposta in due parti che descrivono rispettivamente la variazione imputabile alla localizzazione iniziale e la variazione del mix produttivo regionale. Riguardo a quest'ultima viene proposto un semplice indicatore per evidenziare situazioni critiche in cui valori anomali siano attribuibili alla piccola dimensione iniziale dell'aggregato.

**Keywords:** Growth Components, Omothetic Variables, Location Quotient, Regional Economics.

## 1. Introduction

Suppose that an economic aggregate  $I$  (e.g. the number of employees or the number of businesses) is cross-classified by  $k$  industries and  $m$  areas in two successive times (say  $\alpha$  and  $\omega$ ). We then obtain two two-entry tables  $\{I_{ij}^t \mid i=1, \dots, k \quad j=1, \dots, m\} \quad t = \alpha, \omega$ .

The shift-share analysis (SSA) is a common tool used to analyze the variations occurred to the table's internal entries between  $\alpha$  and  $\omega$ . In short,  $\Delta I_{ij} = I_{ij}^\omega - I_{ij}^\alpha = I_{ij}^\alpha i_{ij}$  is expressed as function of national, industry and area specific growth rates through the identity:

$$\Delta I_{ij} = I_{ij}^\alpha i_{ij} = I_{ij}^\alpha \left\{ i_{00} + (i_{i0} - i_{00}) + \left[ (i_{ij} - i_{0j}) - (i_{i0} - i_{00}) \right] + (i_{0j} - i_{00}) \right\} \quad (1)$$

$$\text{with } i_{00} = \frac{I_{00}^\omega}{I_{00}^\alpha} - 1, \quad i_{+j} = \frac{I_{0j}^\omega}{I_{0j}^\alpha} - 1, \quad i_{i+} = \frac{I_{i0}^\omega}{I_{i0}^\alpha} - 1, \quad i_{ij} = \frac{I_{ij}^\omega}{I_{ij}^\alpha} - 1, \quad i = 1, \dots, k, \quad j = 1, \dots, m.$$

Decomposition (1) allows to identify national ( $N_{ij}$ ), national industry mix ( $MN_{ij}$ ), regional industry mix ( $MR_{ij}$ ) and regional ( $R_{ij}$ ) components of total variation  $\Delta I_{ij}$ :

$$\Delta I_{ij} = I_{ij}^\alpha i_{ij} = N_{ij} + MN_{ij} + MR_{ij} + R_{ij} \quad (2)$$

with

$$N_{ij} = I_{ij}^\alpha i_{00}, \quad MN_{ij} = I_{ij}^\alpha (i_{i0} - i_{00}), \quad MR_{ij} = I_{ij}^\alpha \left\{ (i_{ij} - i_{0j}) - (i_{i0} - i_{00}) \right\}, \quad R_{ij} = I_{ij}^\alpha (i_{0j} - i_{00})$$

The decomposition (2) is due to Arcelus (Arcelus, 1984) and is not unique: in fact many alternative, yet similar, versions of the SSA have been proposed in the literature (see Biffignandi 1991, Ch 2 for a review).

Shift-share results are often used in comparative regional economics analyses to identify disparities and patterns in industry and regional development. For instance, Biffignandi (2005) bases the identification of sixteen economic typologies on the signs of the components in (2) that depend ultimately on the relative magnitude of the national, regional, national and regional industry mix rates  $i_{00}, i_{i0}, i_{0j}, i_{ij}$ . Unfortunately  $i_{ij}$  may be numerically instable: when the aggregate  $I_{ij}^\alpha$  is small, even moderate changes  $I_{ij}^\omega - I_{ij}^\alpha$  may lead to values of  $i_{ij}$  very far from 1 (in the extreme case, when  $I_{ij}^\alpha = 0$ ,  $i_{ij}$  is not defined). In this paper (section 3) we propose a new version of SSA endowed with a modified  $MR_{ij}^\#$  accounting for the initial regional industry mix and depending only on the marginal rates  $i_{00}, i_{i0}, i_{0j}$ . The new version of the SSA involves the omothetic rates  $i_{ij}^O$  (their definition is recalled in Section 2) and a further component  $\Delta QL_{ij}$  which measures the effects of the variation of the location quotient between  $\alpha$  and  $\omega$ .

## 2. Location quotient (LQ) and omothetic variable

Esteban-Marquillas (1972) introduced omothetic variables in the SSA to take into account the effect of initial localization of industries. Omothetic variables  $IO_{ij}^t$  are the quantities one would have observed if there were no interaction between area and industry, i.e. allocation of the aggregate  $I$  to the industries is the same in all areas. To this end, let's define the omothetic variables as:

$$IO_{ij}^t = \frac{I_{i0}^t I_{0j}^t}{I_{00}^t} \quad t = \alpha, \omega \quad (3)$$

Note that  $\sum_{i=1}^k IO_{ij}^t = \sum_{i=1}^k I_{ij}^t = I_{0j}^t$  and  $\sum_{j=1}^m IO_{ij}^t = \sum_{j=1}^m I_{ij}^t = I_{i0}^t$ . Moreover we may observe also that:

$$I_{ij}^t = IO_{ij}^t LQ_{ij}^t, \quad t = \alpha, \omega \quad (4)$$

where  $LQ_{ij}^t = \frac{I_{ij}^t / I_{+j}^t}{I_{i+}^t / I_{++}^t}$  is the LQ for industry  $i$  and area  $j$ , a well known indicator in the regional economics literature.  $LQ_{ij}^t$  provides a measure of the weight of industry  $i$  in area  $j$ , with respect to the weight of the same industry for the whole nation (or reference macro-area).

From (4) it follows that shift share based on the actual measurements  $I_{ij}^\alpha, I_{ij}^\omega$ . leads to the same variation rates to that computed on the omothetic variables whenever the assumption  $LQ_{ij}^\omega / LQ_{ij}^\alpha = 1$  holds.

In principle, we may in fact consider a SSA on the omothetic variables, decomposing the omothetic variation rate  $i_{ij}^O = IO_{ij}^\omega / IO_{ij}^\alpha - 1$ . Note that  $i_{00}^O = i_{00}$  and similarly  $i_{0j}^O = i_{0j}$ ,

$i_{i_0}^O = i_{i_0}$  so the corresponding components of  $\Delta I_{ij}$  can then be obtained multiplying each of the above aggregates by  $LQ_{ij}^\alpha$ . This simple relation (highlighting the role of initial location on the results) does not hold for the regional mix component of variation as  $i_{ij}^O \neq i_{ij}$ . By the way if  $LQ_{ij}^\omega / LQ_{ij}^\alpha = 1$  then  $i_{ij}^O = i_{ij}$ .

### 3. A modification of the Arcelus shift-share analysis

Let's start noting that  $\Delta I_{ij} = I_{ij}^\alpha i_{ij}^O + I_{ij}^\alpha (i_{ij} - i_{ij}^O)$ . This decomposition, explicit function of the omothetic rate  $i_{ij}^O$  allows us to add a further component in the shift-share analysis (1), thus separating the effect of the initial regional industry mix from the variation occurred in the LQs. In fact we may write:

$$\Delta I_{ij} = I_{ij}^\alpha i_{ij} = I_{ij}^\alpha \left\{ i_{00} + (i_{i_0} - i_{00}) + (i_{0j} - i_{00}) + \left[ (i_{ij}^O - i_{0j}) - (i_{i_0} - i_{00}) \right] + (i_{ij} - i_{ij}^O) \right\} \quad (5)$$

which leads to the identification of the following components:

$$\Delta I_{ij} = I_{ij}^\alpha i_{ij} = N_{ij} + MN_{ij} + R_{ij} + MR_{ij}^\# + \Delta LQ_{ij}$$

$$\text{where } MR_{ij}^\# = I_{ij}^\alpha \left\{ (i_{ij}^O - i_{0j}) - (i_{i_0} - i_{00}) \right\} \text{ and } \Delta LQ_{ij} = I_{ij}^\alpha (i_{ij} - i_{ij}^O) = I_{ij}^\omega - I_{ij}^{\omega*} \quad (6)$$

where  $I_{ij}^{\omega*} = QL_{ij}^\alpha IO_{ij}^\omega$ , that corresponds to the delocalized aggregate  $MD_{ij}$  introduced by Biffignandi (1991). This aggregate is the theoretical quantity we would have observed if the count in cell  $(i, j)$  had experienced in time  $\omega$  the same LQ of time  $\alpha$ .

Note that, differently from  $MR_{ij}$ ,  $MR_{ij}^\#$  is a function of the marginal growth rates  $i_{00}$ ,  $i_{i_0}$ ,  $i_{0j}$  only, and is therefore standardized with respect to the changes occurred in the LQ: it gives the portion of variation due to the regional industry mix assuming it remains that of starting time  $\alpha$ . Moreover as function of  $i_{00}$ ,  $i_{i_0}$ ,  $i_{0j}$  is not liable to numerical instability. The remaining variation is then due to the changes in the relative importance of the industry as expressed by the LQ. In particular, relation (6) highlights how the difference between the actual and omothetic rates of variation on the industry by area cell  $(i_{ij} - i_{ij}^O)$  may be expressed as a function of the variation in the LQs between  $\alpha$  and  $\omega$ : Hence, we define the following indicator  $i_{ij}^{LQ}$  for stability analysis of local effects:

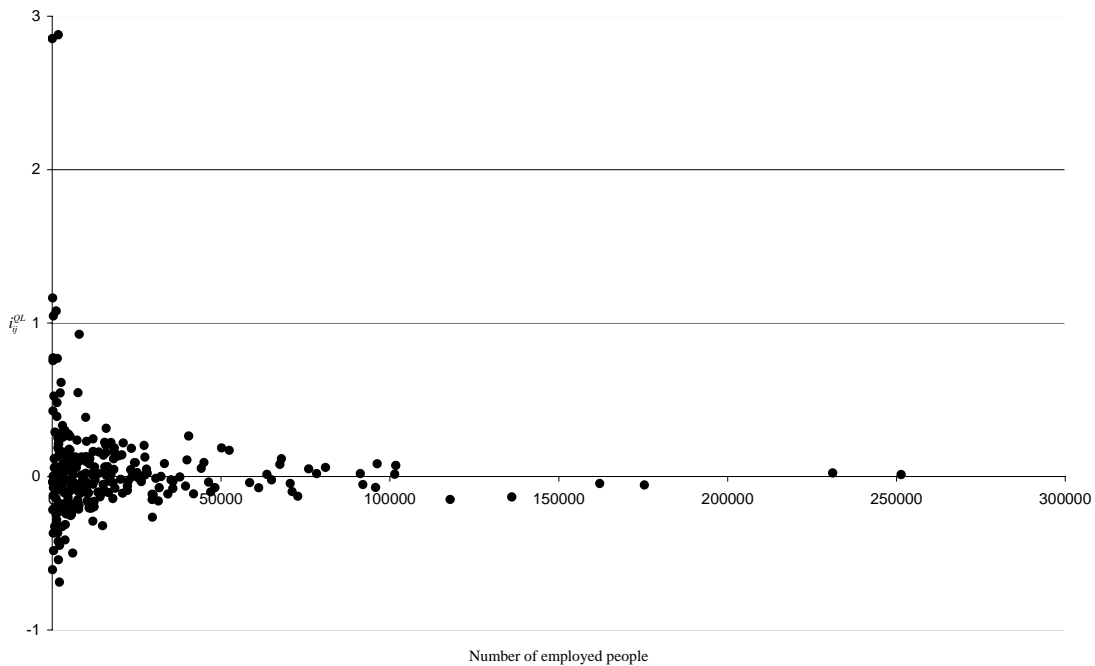
$$\frac{i_{ij} - i_{ij}^O}{i_{ij}^O} = \frac{LQ_{ij}^\omega}{LQ_{ij}^\alpha} - 1 = i_{ij}^{LQ} \quad (7)$$

which can also be used to re-express the  $\Delta LQ_{ij}$  component as  $I_{ij}^\alpha \frac{IO_{ij}^\omega}{IO_{ij}^\alpha} i_{ij}^{OL}$ .

Values of  $i_{ij}^{LQ}$  far from 1 can either be due to a deep change in the relevance of industry  $i$  in area  $j$  or simply to numerical instability: in fact, when the count in cell  $(i, j)$  at time  $\alpha$  is small, even a moderate change in absolute terms may determine an erratic value of  $i_{ij}^{OL}$  which reflects through (7) on the cell specific variation rate  $i_{ij}$ . To illustrate this point consider the following example based on data from the last two Censuses of

Industry and Commerce (1991 and 2001) for which we computed SSA for manufacturing industries and Italy's Administrative Regions. In Figure 1 we plot  $I_{ij}^{1991}$  (number of people employed in industry  $i$  and Region  $j$ ) on the  $x$ -axis against  $i_{ij}^{OL}$  on the  $y$ -axis. We may observe how values of  $i_{ij}^{OL}$  outside the range 0.5 are seldom observed and only for small cells in terms of employed people.

**Figure 1:** The rate  $i_{ij}^{OL}$  versus the initial count (Industry Censuses 1991 and 2001).



The analysis suggests that the rate  $i_{ij} - i_{ij}^O$  and the corresponding component  $\Delta LQ_{ij}$  should be interpreted with care, as they are liable to numerical instability. A visual inspection of a plot such as that displayed in Figure 1 may be of help to identify critical situations. All the remaining component of the proposed version of the SSA are, by construction, not liable not exposed to numerical instability, are easier to interpret from an economic point of view, and can straightforwardly plugged in analyses based on specific growth rates comparisons.

## References

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