MODELLING THE STOCHASTIC BEHAVIOUR OF FOREIGN EXCHANGE AND INTEREST RATES WITH ARCD-M MODELS.

EUMOptFin3 Workshop: The Drivers of Performance of large Financial Institutions

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ABSTRACT

- This paper examines a new class of time series models, the Autoregressive Conditional Density Estimation (ARCD) and the Autoregressive Conditional Density Estimation in Mean (ARCD-M).
- ARCD and ARCD-M allow for variation in the conditional distribution beyond mean & variance.
AGENDA

- Introduction
- ARCH,GARCH,ARCH-M Models Overview
- ARCD – ARCD-M Models Description
- Empirical Analysis: Data Specification, Estimation
- Conclusion
INTRODUCTION

Exchange rates properties are subject to continual interest & research because they have important economic implications for:

1. Currency market equilibrium
2. International trade
3. Capital flows
It is crucial to identify properly the type of heteroskedasticity in the data generating process & the predictive distributions which depend on the knowledge of the correct conditional distribution for the normalised error.

Accurate specification plays an important role in the international finance, particularly in the area of the pricing of foreign currency options and the selection of mean-variance efficient portfolios.
ARCH-GARCH MODELS OVERVIEW

**ARCH** formulation lies in the distinction between the conditional & unconditional second order moments. The general form postulates the conditional variance to be a non-trivial function of the current information set.
ARCH(1) is described by:

\[ y_t = \varepsilon_t (\omega + \alpha \varepsilon_{t-1}^2)^{1/2} \]

\[ E_t(\varepsilon_t) = 0 \]

\[ V_t(\varepsilon_t) = \omega + \alpha \varepsilon_{t-1}^2 \]

Where \( \alpha, \omega > 0 \)
**ARCH** model exhibits constant unconditional variance but non constant conditional variance (time varying). Also the conditional distribution of the normalised error is assumed to be independent of the conditioning information variable.
ARCH(q)

ARCH (1) can be generalised to **ARCH(q)**:

\[ V_t(\varepsilon_t) = \omega + \sum \alpha_i \varepsilon_{t-1}^2 \]

\( \sigma_t \) depends linearly on q lagged values of \( \varepsilon_t^2 \).

- **Disadvantage:** A relatively long lag is called for to avoid problems with negative variance parameters.

- **Result:** Difficulty to estimate models with large number of parameters.
GARCH(p,q)

- *Extension1:* GARCH\((p,q)\) model which allows for both longer memory & a more flexible structure. This model captures thick tailed returns and volatility clustering but are not well suited to capture the leverage effect since the conditional variance is a function only of the magnitudes of the lagged residuals & their signs.
**GARCH(p,q)** process is described by:

\[ V_t (\varepsilon_t) = \omega + \sum a_i \varepsilon_{t-1}^2 + \sum \beta_i v_{t-1} \]

\[ \mathbb{E}(\varepsilon_t) = 0 \]

where \( a, \omega, \beta > 0 \) for all \( i \) so as to ensure the positivity of \( \sigma_t^2 \).

- For \( p=0 \) the process reduces to ARCH(q) & in additional for \( q=0 \) is simply white noise.
**Extension2:** ARCH-M model which allows the conditional variance to affect the mean and hence the expected return on a portfolio. ARCH-M model takes into account that risk premia are not time invariant but vary systematically with agent’s perceptions of underlying uncertainty.
ARCH-M(1) is described by:

\[ y_t = \beta + \delta V_t + \varepsilon_t \]

\[ E_t(\varepsilon_t) = 0 \]

\[ V_t^2 = \omega + \alpha \varepsilon_{t-1}^2 \]

Where \( \alpha, \omega > 0 \)
In the above models the conditional distribution of the normalised error is independent of the conditioning information except its features mean & variance which are time varying, ignoring higher order features such as skewness & degrees of freedom.

- Normalised error is mispecified.
- Negative influence in the accuracy of predictive distributions.
ARCD, ARCD-M Models

- In ARCD model the conditional variance is a function of lagged errors. The suggestion is to select a distribution which depends upon a low dimensional parameter vector which is a time varying function of the conditional variables.

- Bruce Hansen (1994) showed that the conditional density function that fits the data better allowing for time variation in the shape parameters is a generalization of the conditional student’s density called skewed student’s t function.
ARCD-M Model

Here, the idea is to study also the relationship between risk and return by allowing the conditional variance to affect the mean. We parameterize the function $f(y_t|\mu_t, \sigma^2_t, \eta_t)$ taking into account that:

$$y_t = \beta + \delta h_t + e_t h_t^{1/2} \text{ where } \mu_t = \beta + \delta h_t$$

The conditional density function of the normalised error is described by the following equations:
\[ f(z \mid \eta, \lambda) = \begin{cases} 
bc(1 + \left( \frac{1}{\eta-2} \left( \frac{bz + \alpha}{1-\lambda} \right)^2 \right)^{-\frac{\eta+1}{2}} 
\end{cases} \]

where \( z < -\frac{\alpha}{b} \) or

\[ f(z \mid \eta, \lambda) = \begin{cases} 
bc(1 + \left( \frac{1}{\eta-2} \left( \frac{bz + \alpha}{1 + \lambda} \right)^2 \right)^{-\frac{\eta+1}{2}} 
\end{cases} \]

where \( z \geq -\frac{\alpha}{b} \)
where $2 < \eta < \infty$, $-1 < \lambda < 1$ and $a, b, c$ are constants specified as:

- $a = 4\lambda c(\eta - 2)/(\eta - 1)$
- $b^2 = 1 + 3\lambda^2 - a^2$
- $c = \Gamma((\eta + 1)/2)/((\pi(\eta - 2)\Gamma(\eta/2))^{0.5}$

Setting $\lambda = 0$ we take the common student t distribution.
**Constraints:**

1. For the shape parameters $\eta_t$ we use a logistic transformation as a quadratic function of the information set to constrain the degrees of freedom to lie between a lower bound of 2.1 and an upper bound of 30.

2. The variable $\lambda_t$ is bounded between -0.9 & 0.9.
**Note:** For the degrees of freedom the upper bound was selected because student distribution cannot be distinguished from standard normal for any value above 30. The lower bound is 2 because the student distribution does not exist for any value < 2.

Also it is in closed form so as the quasi-likelihood estimation to be feasible & can be easily parameterized so that innovations are mean zero & unit variance.
The normalised error is:
\[ z_t(\theta) = y_t - \mu(\theta,x_t)/\sigma(\theta,x_t) \]
with conditional density function:
\[ g(zl\eta_t) = dP(z_t < zl\eta_t)/dz \]
The conditional log likelihood is maximized using OPTMUM library in Gauss program:
\[ l_t(\theta) = \ln g(z_t(\theta)l\eta_t(\theta)) - \ln \sigma(\theta,x_t) \]
Parameter constancy:

**Nyblom L test:** is an approximate LM test of the null hypothesis that the parameter \( \theta \) is constant against the alternative that \( \theta \) is a martingale process. Here we see if the parameters are stable.
Empirical Analysis: Data Specification, Estimation

**ARCD Model**: data set contains daily quotations of U.S. dollar exchange rates which runs 10 years backward.

**First Step**: We analyze the raw U.S $ cross rates series $S_{it}$ where $i =$ SEK, AUD, JPY, GBP, DKK, CAD, CHF.
1. All currencies have serial correlation at quite extended lags.

2. All currencies have a lot of volatility and especially AUD.

3. The skewness is negative for the most of them implying that their distributions have long left tail except JPY & CHF whose skewness is positive. (No symmetric distribution).
4. The coefficients of kurtosis are smaller than 3 meaning that their distributions are flat platykurtic relative to normal.

5. The possibility of non stationarity is checked by Augmented Dickey Fuller test. The t statistic fails to reject the null hypothesis of a unit root. As a result our series are non stationary as we expected since FX rates exhibit volatility clustering, fat tailed distributions and approximately follow a martingale process.
- **Second step:** We analyse the distributional properties of the data returns taking the first differences of the logarithms of the spot price $p_t$.

1. Durbin Watson test shows that there is no serial correlation (around 2).
2. The skewness is different than zero implying no symmetric distribution.
3. The means are quite small & standard deviations are much less volatile.
4. The kurtosis is larger than 3 meaning that are peaked, leptokurtic relative to normal.

5. There are no unit roots since ADF tests are smaller than the critical values.

6. Our series are stationary since the values move around zero. As a result the non-normality is not due to the non-stationarity.
In our attempt to specify ARCD model we present first the performance of GARCH(1,1) process.

1. Looking at the coefficients the constant term $\omega$ very small for all the currencies.
2. The regression coefficients $\alpha, \beta$ are well significant with t statistics > 1.96.
3. Most of the t-estimated values for the persistence parameter $\beta$ are >5 indicating evidence of strong autocorrelation in the conditional variance.
4. The kurtosis of the unconditional distribution is > 3 explaining the leptokurtosis of the unconditional density.

5. In order to see if the standardised residuals exhibit additional ARCH we apply Lagrange-Multiplier test on the OLS residuals. The null hypothesis of no ARCH effects is rejected.

6. Also all Q statistics for the standardised residuals squared exceed the chi-square 5% critical value. So the variance equation is mispecified.
To assess the fit of the skewed student t model we compare a nonparametric estimate of the density of the standardised residuals with the student t density. For all currencies the density function has similar properties to the student t density.
STANDARDIZED RESIDUALS FIGURE
We calculate the conditional variance, the conditional degrees of freedom, the conditional skewness & Nyblom test values by executing the equivalent Gauss program.

Comparing the likelihoods, the likelihood ratio test is statistically significant meaning that the time-varying shape parameters are statistically significant.
1. For AUD, CAD, & CHF the estimates of the degrees of freedom are close to 5.
2. For DKK, GBP, SEK, JPY most of estimates are close to 4.
3. For all currencies the sequence of the skewness $\lambda_t$ is near zero with the density becoming cond. skewed after large innovations.
CONDITIONAL SKEWED PARAMETER FIGURE

SEK

DKK

CAD

AUD
Finally, we examine the shape parameters stability applying Nyblom L test. For the conditional variances, the cond. degrees of freedom & the cond. skewness the test shows that these above parameters are constant.
RESULTS

- ARCD model fits the data better & with higher degree of precision achieving a full specification & a high qualitative accuracy of predictive distributions.

- The main finding of this application is that the shape parameters of the conditional densities are statistically significant at 1% & 5% level.
ARCD –M RESULTS

- **ARCD-M Model**: data set contains daily quotations of U.S. Treasury Securities which runs 10 years backward.
- We analyze the raw U.S interest rates series $S_{it}$ where $i=1$ month, 6Month, 1-year, 5-years, 10-years.
- The excess holding yield $y_t$ is calculated as
  \[ y_t = \frac{(1+R_t)^2}{1+r_{t+1}} - (1+r_t) \]
  where $r_t$ is the instantaneous yield rate.
Comparing the likelihoods for GARCH and ARCD-M we see for all the series, that the fit of the model is a dramatic improvement over the Gaussian, with the log-likelihood changing overall by 11.5.

For all the data the estimates of the degrees of freedom are close to 5 which implies fairly fat tails.

Finally, we examine the shape parameters stability applying Nyblom L test. For the conditional variances, the cond. degrees of freedom & the cond. skewness the test shows that the above parameters are constant.
CONCLUSION

- ARCD and ARCD-M models fit the data better & with higher degree of precision achieving a full specification & a high qualitative accuracy of predictive distributions.

- The main finding of this application is that the shape parameters of the conditional densities are statistically significant at 5% level.

- The higher order features play important role in the specification of their density & in any case should not be ignored.
Further research:

- The forecasting power of the above models have to be examined.

- ARCD and ARCD-M models can be extended in the context of options pricing, where full specification is important, & the price is determined by not just the cond. mean & variance.